RESEARCH MEMORANDUM

DETERMINATION OF TRANSIENT SKIN TEMPERATURE OF CONICAL BODIES DURING SHORT-TIME, HIGH-SPEED FLIGHT

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SUMMARY

A short and simple method is presented for the determination of transient skin temperature of conical bodies for short-time, high-speed flight. A differential equation is established for this purpose, giving the fundamental relations between the transient skin temperature and the flight history. For the heat-transfer coefficient and boundary-layer temperature, which are needed in the differential equation, Eber's experimental results for conical bodies under supersonic conditions are adapted and summarized in a convenient way. The method is applied first for flight at constant altitude to illustrate the effect of acceleration on transient skin temperature. It is then applied to arbitrary flight. Several examples are given; for one example measured data are available and are in good agreement with the calculations.

INTRODUCTION

When air flows over a body the air immediately adjacent to the body is brought to rest by skin friction. As a result the air is heated to a higher temperature and hence there is heat exchange between the air and the skin. This phenomenon is generally termed as "aerodynamic heating."

At high speed the temperature increase of the air is very large and the aerodynamic heating problem becomes of great concern to designers. The problem is related to the characteristics of the boundary layer and the local heat-transfer coefficient. In reference 1 Wood gave a method for the determination of the skin temperature at supersonic speed, using formulas for heat-transfer coefficient and boundary-layer temperature, derived for flat plates at subsonic speed. In reference 2 Scherrer made a more theoretical approach for the determination of skin temperature of a body of revolution at supersonic flight. Both papers, however, dealt with equilibrium skin temperature for steady flight conditions at constant altitude.

In several German papers (reference 3, for instance) it was shown that, for a short-time flight during which the speed and altitude vary with time, the transient skin temperature may be considerably lower than the equilibrium skin temperature. In the present report, therefore, emphasis is given to the transient skin temperature, rather than the equilibrium skin temperature. A differential equation is presented for this purpose, and for the heat-transfer coefficient and boundary-layer temperature needed in the differential equation, Eber's experimental results (reference 4) for conical bodies under supersonic conditions are adapted and summarized in a convenient form for immediate application. If, however, better experimental data become available, they can be adapted readily to the present method.

To show the effect of acceleration on skin temperature the simpler problem of flight at constant altitude is treated first. More general flights are then discussed with several examples. In one example, data obtained from the Naval Research Laboratory, Washington, D.C., giving skin temperatures for V2 missile number 21 fired on March 7, 1947 at White Sands, N. Mex., in connection with upper atmosphere research (reference 5), are used for comparison with the calculated results.

SYMBOLS

a	acceleration, ft/sec^2
c	recovery factor
c_p	specific heat of air at constant pressure, Btu/lb/$^{\circ}$F
c_s	specific heat of skin material, Btu/lb/$^{\circ}$F
f	function of T_{sk} and implicitly of t (see equation (16))
g	acceleration due to gravity, taken as 32.2 ft/sec^2
h	heat-transfer coefficient, Btu/(sec)(sq ft)($^{\circ}$F)
h'	reference heat-transfer coefficient corresponding to $\beta = \pi/6$ (or 30°), $l = 1$ ft and H = sea level, Btu/(sec)(sq ft)($^{\circ}$F)
k	thermal conductivity of air, Btu/(sec)(ft)($^{\circ}$F)
l	characteristic length, ft
t	time, sec

t_a	time when certain velocity is reached by uniformly accelerated body starting from rest, sec
F_β	correction factor for nose angle (see equation (14))
F_l	correction factor for characteristic length (see equation (14))
F_H	correction factor for altitude (see equation (14))
G	heat absorption capacity of skin, Btu/sq ft/°F
H	altitude, ft
J	mechanical equivalent of heat, taken as 778 ft-lb/Btu
Q_1	heat flowing into skin due to skin friction, Btu/sec/sq ft
Q_2	heat lost through radiation, Btu/sec/sq ft
R	Reynolds number
T_1	free-stream temperature of air, °F abs.
T_a	skin temperature at time t_a, °F abs.
T_e	equilibrium skin temperature, °F abs.
T_i	initial temperature, °F abs.
T_{BL}	boundary-layer temperature, °F abs.
T_{sk}	skin temperature, °F abs.
T_{ST}	stagnation temperature, °F abs.
V_1	velocity of flight, ft/sec
W	specific weight of skin material, lb/cu ft
β	total apex angle of conical nose, radians
ϵ	emissivity, assumed to be 0.4 in numerical examples of this report ($\epsilon = 1$ for perfect black body)

ρ_1 free-stream density of air, lb/cu ft

ρ_{SL} free-stream density of air at sea level, taken as 0.07657 lb/cu ft

μ coefficient of viscosity of air, lb/sec/ft

τ skin thickness, ft

ANALYSIS

Fundamental Equations

The skin temperature of a body can be determined from a consideration of the amount of heat flowing into the skin and that lost by the skin. The balance of the two is the heat absorbed or given up by the skin from which the change in skin temperature can be calculated.

Heat flowing into skin.- For heat flowing into the skin during a short-time, high-speed flight, the most important factor to be considered is the "aerodynamic heating." Experimental results indicate that the heat flowing into the skin due to skin friction Q_1 can be determined from the following empirical formula:

$$Q_1 = h\left(T_{BL} - T_{sk}\right) \qquad \text{Btu/sec/sq ft} \tag{1}$$

where the boundary-layer temperature T_{BL} and the heat-transfer coefficient h have been studied experimentally by Eber (reference 4) and will be discussed later.

Heat received by the skin from other sources, like solar radiation, radiation from surrounding atmosphere, heat from interior or other parts of the body, and so forth, are not considered in this report.

Heat lost by the skin.- The heat lost by the skin can be (1) heat loss through radiation, (2) heat loss through artificial cooling, (3) heat loss to interior or other parts of the body. The heat loss through radiation Q_2 can be determined from the Stefan-Boltzmann formula:

$$Q_2 = 4.8 \times 10^{-13}\, \epsilon T_{sk}{}^4 \qquad \text{Btu/sec/sq ft} \tag{2}$$

where the value of the emissivity ϵ depends on the surface condition of the skin. For a perfect black body, $\epsilon = 1$.

The heat loss other than radiation to the exterior will not be considered in this report.

Heat balance equation.– Equation (1) gives the rate of heat flow into the skin and equation (2) gives the rate of heat loss by the skin. The difference of the two $\left(Q_1 - Q_2\right)$ is the heat left to heat the skin, Btu/sq ft/sec. During an interval dt, the total heat to be absorbed by the skin is therefore

$$\left(Q_1 - Q_2\right) dt \qquad \text{Btu/sq ft}$$

If the temperature rise of the skin during this interval dt is dT_{sk}, the heat absorbed by the skin can also be expressed by

$$G\ dT_{sk} \qquad \text{Btu/sq ft}$$

where G, the heat absorption capacity of the skin, Btu/sq ft/°F, is the product of the specific heat of the skin material c_s, the specific weight of the skin material W, and the skin thickness τ. From data in references 6 and 7, representative values of G for steel and alloys of aluminum and magnesium at 520° F are obtained and plotted against skin thickness in figure 1. At higher temperature the values of G are increased at the rate of $0.256\tau'$ per 100° F for steel and $0.0624\tau'$ per 100° F for aluminum where τ' is the skin thickness in inches. For magnesium alloy the temperature effect can be neglected.

Equating the preceding two expressions for the heat absorbed by the skin and substituting Q_1 and Q_2 from equations (1) and (2) gives

$$G\frac{dT_{sk}}{dt} + hT_{sk} + 4.8 \times 10^{-13}\epsilon T_{sk}{}^{4} = hT_{BL} \tag{3}$$

which is the basic equation for transient skin temperature.

The simplified equation.– Equation (3) is a nonlinear differential equation with variable coefficients. A simplified equation can be obtained if the radiation loss can be neglected. The simplified equation is

$$G\frac{dT_{sk}}{dt} + hT_{sk} = hT_{BL} \tag{4}$$

As will be shown later (example 3) the radiation loss ordinarily contributes only a small portion to the transient skin temperature for short-time, high-speed flight. For such flights the use of equation (4) will not introduce any appreciable error, while a great deal of time and labor can be saved in the computation work.

Equilibrium temperature.- If a body flies with a constant speed at a constant altitude for a sufficiently long time, the skin is heated to a temperature such that the heat flowing into the skin per second is for practical purposes the same as the heat lost, and the skin temperature becomes constant. The skin temperature then is called the equilibrium temperature T_e, corresponding to that speed and altitude. The equilibrium temperature T_e can be determined from equation (3) by dropping out the first term since $\frac{dT_{sk}}{dt} = 0$. Thus,

$$4.8 \times 10^{-13}\epsilon T_e^{\ 4} + hT_e = hT_{BL} \tag{5}$$

If the radiation loss can be neglected, the equilibrium skin temperature will be the same as the boundary-layer temperature.

In equations (3), (4), and (5) the two parameters h and T_{BL} are needed before the equations can be solved. In the following, Eber's experimental data on these two parameters will be discussed and summarized.

Eber's Experimental Results

In 1941 Eber (reference 4) made a series of wind-tunnel tests on cones of various vertex angles to determine the boundary-layer temperature T_{BL} and heat-transfer coefficient h at high speeds. The results he obtained are adapted in this report for the determination of skin temperature for conical bodies in flight. These results will be discussed in the following and will be summarized in a convenient form with certain simplifications.

Boundary-layer temperature, T_{BL}.- When the air stream is brought to rest isentropically, the temperature of the air is called stagnation temperature. When the air is brought to rest by skin friction, the process is usually not isentropic and the temperature of the air is lower than the stagnation temperature. In the latter case, if there is no heat flow across the skin, the temperature of the air immediately adjacent to the skin is called the boundary-layer temperature T_{BL} and is found to be closely related to the stagnation temperature. In the actual case where there is heat exchange between the air and the skin, the boundary-layer temperature T_{BL} is only an artificial term used in the empirical equation (1) and may not necessarily be realized at any point in the actual boundary layer.

The stagnation temperature T_{ST} can be calculated theoretically from the following statement of Bernoulli's equation:

$$\int_{V_1}^{0} V\,dV + \int_{T_1}^{T_{ST}} Jgc_p\,dT = 0 \tag{6}$$

where V_1 and T_1 are the velocity and the temperature of the free air stream. Values of T_1 at various altitudes are given in reference 8 and are replotted in figure 2 of this paper.

For constant value of c_p, equation (6) becomes

$$T_{ST} - T_1 = \frac{1}{2}\frac{V_1^2}{Jgc_p} \tag{7}$$

For the actual case, however, c_p varies with temperature. It has been pointed out by Wood (reference 1) that at high velocity a correction for variable c_p will lower the stagnation temperature considerably (about 20 percent lower at Mach number 8). An exact solution of Bernoulli's equation for variable c_p will give a set of stagnation temperature curves for various altitudes and Mach numbers. In this report, however, it is chosen to plot the stagnation temperature rise $\left(T_{ST} - T_1\right)$ against free-stream velocity V_1, instead of the stagnation temperature against Mach number. The result is a single curve for all altitudes. This curve, for which the derivation is given in appendix A, is plotted in figure 3, together with the stagnation temperature rise for $c_p = 0.24$ Btu/lb/°F. For low speeds, no correction for variable c_p is necessary.

The boundary-layer temperature T_{BL} is related to the stagnation temperature by an empirical factor c, called recovery factor. The recovery factor c is defined as

$$c = \frac{T_{BL} - T_1}{T_{ST} - T_1} \tag{8}$$

Eber's experimental results show that c varies with the total vertex angle β of the cone but is practically independent of velocity. The variation of c with β is not very large. For β ranging from 20° to 50°, an average value $c = 0.89$ can be used with a maximum error of about 2 percent. Therefore,

$$\left(T_{BL} - T_1\right) = 0.89\left(T_{ST} - T_1\right) \tag{9}$$

Since the stagnation temperature rise $\left(T_{ST} - T_1\right)$ can be taken as a function of V_1 only (fig. 3), the boundary-layer-temperature rise can also be taken as a function of V_1; that is, a single curve for all altitudes. This curve is shown in figure 4. For $c_p = 0.24$ Btu/lb/°F, the boundary-layer-temperature rise can be given by the following expression:

$$T_{BL} - T_1 = 74.0\left(\frac{V_1}{1000}\right)^2 \tag{10}$$

Heat-transfer coefficient, h.- The heat-transfer coefficient h can be determined from Eber's empirical formula which is taken from reference 4:

$$h = \left(0.0071 + 0.0154\sqrt{\beta}\right)\frac{k}{l}(R)^{0.8} \tag{11}$$

where the Reynolds number R, as defined by Eber, is equal to $\left(\rho_1 V_1 l/\mu\right)$. Equation (11) can be rewritten as

$$h = (0.0071 + 0.0154\sqrt{\beta})\left(\frac{1}{l^{0.2}}\right)(\rho_1)^{0.8}\left[\frac{k}{\mu^{0.8}}(V_1)^{0.8}\right] \qquad (12)$$

In the derivation of equation (11), Eber used the boundary-layer temperature as the reference temperature for values of k and μ, the thermal conductivity and coefficient of viscosity of air, respectively. For the characteristic length l, Eber used the entire length of the surface of the cone. Although there are different opinions regarding what is the correct length to be used as characteristic length, when Eber's experimental results are applied to actual flight conditions (for instance, in reference 3 Kraus and Hermann used half the total length of the cone surface as characteristic length) Eber's original definition will be followed in this paper. The value of $\bar{h}$ thus obtained represents an average value, instead of local value, of the heat-transfer coefficient.

A simplified method for the evaluation of the factor $\left(k/\mu^{0.8}\right)$ is now given. For a given velocity and altitude, the boundary-layer temperature can be determined from figures 2 and 4, and values of k and μ corresponding to this temperature can be obtained from reference 9 for temperatures below 2400° F absolute. This was done for various velocities at three different altitudes (sea level, 100,000 ft and 190,000 ft) and the ratio $k/\mu^{0.8}$ was calculated and plotted against velocity in figure 5. For boundary-layer temperature higher than 2400° F absolute the curve is shown dotted and is obtained by extrapolation.

In figure 5 the two curves corresponding to 100,000 feet and 190,000 feet form two limiting values of $k/\mu^{0.8}$. For any other altitudes from sea level to 370,000 feet, approximately, values of $k/\mu^{0.8}$ against velocity will fall within these two limiting curves. Since both limiting curves do not differ very much from the sea-level curve, it is justified to use the sea-level curve for all altitudes up to 370,000 feet. Therefore, $k/\mu^{0.8}$ is a function of velocity only.

In connection with this simplification, it should be kept in mind that values of k and μ at high temperature are obtained by extrapolation and any effects of change of air composition at high altitudes on k and μ are not considered.

Equation (12) is now reduced to four factors which are functions of β, l, altitude and velocity, respectively. For convenience of computation, a reference heat-transfer coefficient h', corresponding to $\beta = \frac{\pi}{6}$ (or 30°), $l = 1$ foot, and at sea level, is computed from equation (12) and the results are plotted in figure 6. This reference heat-transfer coefficient h' is a function of velocity only. For other nose angles, characteristic lengths and altitudes, the heat-transfer coefficient h is simply the reference heat-transfer coefficient h' multiplied by the three correction factors F_β, F_l, and F_H. Thus,

$$h = F_\beta F_l F_H h' \tag{13}$$

where

$$\left.\begin{aligned} F_\beta &= \frac{0.0071 + 0.0154\sqrt{\beta}}{0.0071 + 0.0154\sqrt{\pi/6}} \\ F_l &= \frac{1}{l^{0.2}} \\ F_H &= \left(\frac{\rho_1}{\rho_{SL}}\right)^{0.8} \end{aligned}\right\} \tag{14}$$

Values of F_β, F_l, and F_H are given in figures 7, 8, and 9. Values of (ρ_1/ρ_{SL}), where ρ_{SL} is the free-stream air density at sea level, were taken from the NACA standard atmosphere table (reference 10) for altitudes below 65,000 feet and from tables V(a) and V(b) of reference 8 for altitudes above 65,000 feet.

Application of Eber's results.- In the foregoing, Eber's experimental results on T_{BL} and h are represented by simple curves as functions of velocity and altitude. For any prescribed flight path where the velocity and altitude are given as functions of time, values of T_{BL} and h can be expressed as functions of time. Table 1 is prepared for this purpose and the operations in table 1 are self-explanatory. Knowing T_{BL} and h as functions of time, one can solve equation (3) or (4) for the transient skin temperature, and equation (5) for the equilibrium skin temperature.

Certain facts should be borne in mind, however, in the application of Eber's work, particularly to high-altitude flight conditions. First, Eber's experimental results were obtained over a limited range of Reynolds number, 2×10^5 to 2×10^6; for flight conditions where the actual Reynolds number falls outside of this range (which is usually the case for flight at high altitudes), extrapolation is necessary. Second, Eber's experiments were carried out at low altitudes. At high altitudes where the air density is very low, the flight may enter into the slip-flow domain where the recovery factor, and possibly the heat-transfer coefficient, may be greatly affected. (For illustrations, calculations were made to determine the Mach numbers and Reynolds numbers from the flight trajectory of the V2 missile used in example 1 of this report. The results of these calculations are indicated by points in figure 10, where the curve dividing the two domains is taken from reference 11. For this particular example the flight enters the slip-flow domain as soon as an altitude of approximately 130,000 feet is reached.) Third, in the evaluation of k and μ, the effects of change of air composition at high altitudes are neglected and values of k and μ at temperature greater than 2400° F absolute are obtained by extrapolation. Finally, the properties of the atmosphere at high altitudes are taken from the tentative tables of reference 8. With all these uncertainties the application of Eber's results to high altitudes may introduce a large percentage error. However, since the heat-transfer coefficient at high altitude is small, the effect on skin temperature is not large.

Solutions of Equations

Solution of equation (3).- Equation (3) is a nonlinear equation with variable coefficients, of the first order but the fourth degree. If equation (3) is written in the following form

$$\frac{dT_{sk}}{dt} = f\left(T_{sk}, t\right) \tag{15}$$

where

$$f = \frac{h}{G}T_{BL} - \frac{h}{G}T_{sk} - \frac{4.8 \times 10^{-13}\epsilon}{G}T_{sk}^{\ 4} \tag{16}$$

it is readily recognizable that equation (15) can be solved conveniently by Runge and Kutta's numerical method (reference 12) which is summarized in appendix B. In table 2 Runge and Kutta's method is arranged in a suitable way for the solution of equation (15). A convenient time interval Δt is first chosen. The smaller Δt is, the more accurate the results will be. The operation of this table starts on the first line, proceeding from left to right, and then the succeeding lines. The function f, corresponding to t and T_{sk} at its left, can be obtained from equation (16). The final results of table 2 give the skin temperature at the end of the interval Δt. If the table is repeated, the skin temperature at the end of $2\Delta t$, $3\Delta t, \ldots$, and so forth, can be obtained. If the time interval chosen is not small, a correction can be made as explained in appendix B, where an illustrative example is also given.

Solution of equation (4).- Equation (4) is a linear differential equation of the first order provided the heat absorption capacity G is considered to be independent of skin temperature. The general solution is

$$T_{sk} = e^{-\int_0^t \frac{h}{G} dt}\left(\int_0^t \frac{h}{G} T_{BL} e^{\int_0^t \frac{h}{G} dt} dt + D\right) \tag{17}$$

where D, the constant of integration, can be determined from the initial condition

$$T_{sk} = T_i \quad \text{at} \quad t = 0$$

The parameters h and T_{BL} in equation (17) usually cannot be expressed in simple analytical terms. Therefore, equation (17) has to be integrated numerically. Table 3 is provided for this purpose. The time and labor required for carrying out the computations in table 3 are much less than those required for table 2.

Solution of equation (5).- Equation (5) is a simple quartic algebraic equation. By use of Eber's experimental results on h and T_{BL}, equation (5) can be solved for the equilibrium temperature T_e for any altitude and velocity combination. For illustrative purposes, the variation of equilibrium temperature with velocity for several altitudes is shown in figure 11. The value of ϵ used is 0.4.

RESULTS AND DISCUSSION

The method for the calculation of transient skin temperature, as discussed in the preceding sections, is applied first to flight at constant altitude to illustrate the influence of acceleration on skin temperature, and then to arbitrary flight conditions.

Flight at Constant Altitude

For illustration of the effect of acceleration, the simplified equation (4) will be used to determine the skin temperature for the following example.

Assume a body, starting from rest, is accelerated uniformly to 5000 feet per second. It then maintains this speed until the equilibrium temperature is reached. Subsequently, the body decelerates uniformly to zero velocity again. How does the skin temperature change with time during these three periods of flight? The body has a conical nose angle of 30°, a conical surface length of 1 foot, a skin heat-absorption capacity $G = 0.6$, and is traveling at a constant altitude of 50,000 feet.

Period of uniform acceleration.- The velocity of the body at any instant during this period is given by the following equation

$$V_1 = at$$

where a = acceleration, feet per second^2. The initial condition is $T_{sk} = T_1$ at $t = 0$.

Since the velocity is given as function of time, T_{BL} and h can be determined from table 1 and the skin temperature from table 3, all as functions of time. The results are given in figure 12 for five different values of acceleration: $a = 2g$, $5g$, $10g$, $50g$, and ∞ (for infinite acceleration the temperature-time curve is but a single point). The dotted curve in figure 12 gives the skin temperature T_a at the time t_a when the body reaches 5000 feet per second. The larger the acceleration is, the lower the skin temperature will be.

In figure 13, the skin temperatures for $a = 2g$ and $10g$ are plotted against V_1, together with the curve showing the variation of equilibrium temperature with velocity. Since radiation loss is neglected in this case, the equilibrium temperature is the same as the boundary-layer temperature. The temperature difference between the "transient" and the "equilibrium" curves is the "temperature lag," which is greater for larger accelerations, as shown in figure 13.

Period of constant velocity.- If the body maintains its speed at 5000 feet per second after it reaches this velocity, h/G and T_{BL} are no longer functions of time, and the solution of equation (17) becomes

$$T_{sk} = T_{BL} + De^{-(h/G)t} \qquad \text{for} \quad t \geqq t_a \tag{18}$$

where D, the constant of integration, is determined from the condition

$$T_{sk} = T_a \quad \text{when} \quad t = t_a$$

Here T_a, the skin temperature at time when the body first reached the velocity of 5000 feet per second is different for different accelerations.

Equation (18) is solved for the five different accelerations $a = 2g$, $5g$, $10g$, $50g$, and ∞. Results are plotted in figure 14. (The temperature curves before the body reaches 5000 ft/sec are taken from fig. 12.) The skin temperature approaches exponentially the equilibrium temperature T_e, in this case equal to T_{BL} corresponding to 5000 feet per second at the altitude of 50,000 feet.

Period of uniform deceleration.– If after the body maintains 5000 feet per second for a long time, it starts to decelerate, the velocity at any instant is given by

$$V_1 = 5000 + at$$

where t starts when the body begins to decelerate. The initial condition is

$$T_{sk} = T_{BL} \quad \text{at} \quad t = 0$$

Table 1 and table 3 again can be used to solve equation (17) and the results are plotted against velocity in figure 15 for two different decelerations $a = -2g$, $-10g$. Again the equilibrium temperature curve is also given. The transient skin temperature in the case of deceleration is higher than the equilibrium temperature throughout the velocity range. The temperature lag is therefore on the adverse side.

General Flight Conditions

In the general case, the body is changing its altitude as well as its velocity. Three examples are given in the following. In all cases, the simplified equation (4) is used for the calculation of transient skin temperatures. In example (3) the more accurate method is also used and the relative importance of radiation loss is discussed.

Example 1.– The V2 missile number 21 fired on March 7, 1947 at White Sands, N. Mex., had a conical vertex angle of 26°, a conical surface length of approximately 7 feet and a skin of 0.109-inch steel after a certain distance from the nose. The above specifications and the flight path of V2 as shown in figure 16 were obtained from the Naval Research Laboratory, Washington, D.C. The boundary-layer temperature and the heat-transfer coefficient are determined by means of table 1 and are plotted against time in figure 17. The skin absorption capacity G is obtained from figure 1 to be 0.476 corresponding to 520° F absolute, which is conservative. The initial skin temperature is known to be 546° F absolute. Table 3 is used to determine the skin temperatures and the results are plotted against time in figure 18.

In figure 18 the measured skin temperature for the steel skin of the same missile is also shown. The measured data are obtained from the Naval Research Laboratory. The agreement between the calculated and measured results is good.

Example 2.– An arbitrary velocity and altitude diagram is assumed, as shown in figure 19, including descending path as well as ascending. The missile is assumed to have a nose angle of 30°, a characteristic length of 4 feet, and a skin heat-absorption capacity of 0.6 Btu/sq ft/°F. The boundary-layer temperature T_{BL} and heat-transfer coefficient h are determined by use of table 1 and are plotted against time in figure 20. Table 3 is then used to calculate the skin temperatures. The results are plotted in figure 21. The skin temperature during descent becomes higher and higher because of the greater density at the lower altitudes and the higher velocity as it comes down.

Example 3.– To determine the relative importance of the radiation loss, the calculation of skin temperature for the missile in example 2 for the first 80 seconds is repeated except now the more accurate method is used where the radiation loss is not neglected (the emissivity ϵ is assumed to be 0.4). Table 2 is used for this purpose and the results are plotted in figure 22 together with the results obtained from example 2 where the radiation loss is neglected. The discrepancy between the two is very small, approximately 3° F.

In fact, for most missiles the radiation loss plays only a small part in the determination of skin temperature and the simplified method can be used to great advantage in saving time and labor. Unfortunately, there is no simple criterion to predict when the radiation loss can be neglected. Generally speaking, if the skin heat capacity is not too small, say not below 0.5 Btu/sq ft/°F, the skin temperature is not expected to be higher than 1000° F absolute and if the radiation loss is not the dominating factor for a long period of time, the radiation loss can be neglected. If the final skin temperature is completely unknown, it is advantageous to start with the simplified method. From the maximum skin temperature obtained, an estimate of the radiation loss and its effect on the skin temperature can be quickly made. For instance, the maximum skin temperature of the missile of example 2 is about 615° F absolute. Conservatively assume this is the skin temperature for the entire period of 80 seconds. The change of skin temperature due to radiation loss during this period is then

$$\Delta T_{sk} = \frac{1}{G}\left(4.8 \times 10^{-13}\epsilon T_{sk}^{4}\right) t$$

$$= \frac{1}{0.6}\left(4.8 \times 10^{-13} \times 0.4 \times 615^{4}\right) \times 80$$

$$= 3.7^{\circ}\ \mathrm{F}$$

which is negligible. A similar calculation for example 1 indicates that for the first 65 seconds of flight the radiation loss affects the maximum skin temperature approximately 6° F.

For flight conditions where the radiation loss is the dominating factor for a long period, as from 80th second to 220th second in example 2, the radiation loss should be investigated. During this period, h is zero and equation (3) becomes

$$G\frac{dT_{sk}}{dt} + 4.8 \times 10^{-13}\epsilon T_{sk}^{4} = 0$$

The general solution is

$$T_{sk} = \left(14.4 \times 10^{-13}\epsilon \frac{t}{G} + D\right)^{-1/3} \tag{19}$$

where D, the constant of integration, can be determined from the condition

$$T_{sk} = \left(T_{sk}\right)_{80} \quad \text{when} \quad t = 80$$

where $\left(T_{sk}\right)_{80}$ is the skin temperature at the 80th second. At end of 220th second the skin temperature can be calculated from equation (19) and is found to be lowered only 6° F.

CONCLUDING REMARKS

1. A differential equation taking into account aerodynamic heating and body radiation is established for the calculation of transient skin temperature for any prescribed flight history. Runge and Kutta's numerical method is recommended for the solution of the differential equation.

2. A simplified differential equation which neglects body radiation is also given, and can be used in many cases to great advantage in saving both time and labor.

3. Eber's experimental results on the boundary-layer temperature and heat-transfer coefficient, to be used in the differential equation, are summarized in a convenient way for immediate application. Tables and charts are also provided to facilitate the solution of the differential equation.

4. The calculated skin temperature for a V2 missile is in good agreement with the measured data.

5. The heat-absorption capacity of the skin has an important influence on transient skin temperature. The heat-absorption capacity is greater and consequently the temperature lag is larger if (a) the skin is thicker, (b) the material is denser, or (c) the specific heat is higher.

6. When the air is heating the skin the temperature lag due to the heat capacity of the skin is in the favorable direction; that is, tends to lower the skin temperature. When the air is cooling the skin, the temperature lag is in the adverse direction; that is, tends to keep the skin at high temperature.

7. Eber's experimental work was conducted under certain limited conditions (short testing time, small temperature difference, limited Reynolds number, and so forth). More refined experimental values for a wider range are desirable.

8. Because the atmospheric properties at extremely high altitudes as given in reference 8 are tentative, and also because at high altitudes the flight actually enters the "slip-flow" domain, a closer investigation of the problem at high altitudes is needed.

9. For more accurate results, investigations of heat exchanges other than those considered in this paper are necessary.

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APPENDIX A

STAGNATION TEMPERATURE RISE FOR VARIABLE c_p

In the following, a single curve for all altitudes is obtained for the calculation of stagnation temperature rise at various velocities V_1 when the specific heat c_p is considered to be a function of temperature. The Bernoulli equation can be written as

$$V\,dV + Jgc_p\,dT = 0 \tag{A1}$$

Integrate equation (A1) between state 1, the free-stream condition, and state 2, where the velocity is zero.

$$-\int_{V_1}^{0} V\,dV = \int_{T_1}^{T_{ST}} Jgc_p\,dT$$

or

$$\frac{{V_1}^2}{2} = Jg\int_{T_1}^{T_{ST}} c_p\,dT \tag{A2}$$

Values of the integral in equation (A2) can be obtained from table 1, reference 9 (see also page 58, reference 9) for given values of T_1 and T_{ST}, and the velocity V_1 can be computed. A set of curves can thus be obtained.

However, if it is chosen to plot $\left(T_{ST} - T_1\right)$ against V_1 with T_1 as parameter, the set of curves will practically fall into one single curve for all values of T_1 ranging from 392° F absolute to 630° F absolute, corresponding to the minimum and maximum free-stream temperature for altitudes from sea level to about 370,000 feet. In figure 23 two curves are shown, representing the two extreme conditions. It is justified, therefore, to use a single curve for the calculation of stagnation temperature rise.

The boundary-layer-temperature rise $(T_{BL} - T_1)$ can be obtained by multiplying the stagnation temperature rise by the recovery factor c. The result will be a single curve similar to the curve of stagnation temperature rise but with all the ordinates decreased by the ratio c. Figure 4 shows the curve of boundary-layer-temperature rise at various speeds V_1, corresponding to $c = 0.89$.

APPENDIX B

RUNGE AND KUTTA'S METHOD OF NUMERICAL INTEGRATION

Runge and Kutta's method of numerical integration can be applied to differential equations of the following type, provided that the initial condition is known.

$$\frac{dy}{dx} = f(x,y) \tag{B1}$$

The initial condition is

$$y = y_o \quad \text{when} \quad x = x_o \tag{B2}$$

The derivation of Runge and Kutta's method can be found in reference 12. The following table is provided for the calculation.

x	y	f	fx(Δx)	Operation
x_o	y_o	$f\left(x_o,y_o\right)$	q_1	$\frac{1}{2}\left(q_1+q_4\right)$ =
$x_o+\frac{\Delta x}{2}$	$y_o+\frac{1}{2}q_1$	$f\left(x_o+\frac{\Delta x}{2},y_o+\frac{1}{2}q_1\right)$	q_2	q_2+q_3 =
$x_o+\frac{\Delta x}{2}$	$y_o+\frac{1}{2}q_2$	$f\left(x_o+\frac{\Delta x}{2},y_o+\frac{1}{2}q_2\right)$	q_3	Sum =
$x_o+\Delta x$	y_o+q_3	$f\left(x_o+\Delta x,y_o+q_3\right)$	q_4	$q=\frac{1}{3}$ Sum =

$$x_1 = x_o + \Delta x \qquad y_1 = y_o + q$$

For convenience of application to the present problem, Runge and Kutta's method is arranged in the form of table 2 of this paper. A suitable interval Δt should be chosen. Starting from the pair of initial values t_o and T_o, one can obtain the values t' and T' at the end of the interval Δt, by carrying out the operations in table 2. The operations proceed from left to right of the first line and then the succeeding lines. The function $f(t,T_{sk})$, corresponding to t and T to its left on the same line, can be obtained from equation (16). If the pair of values t' and T' are used as initial values and the table repeated, another pair of values t" and T" can be obtained, corresponding to the skin temperature at the end of $2\Delta t$. Table 2 can be repeatedly used in this manner until the desired values of T are obtained.

For illustration, table 2 will be used to calculate the skin temperature of the missile in example 2. Assume the skin temperature at the end of 20th second is known to be 519° F absolute, and the skin temperature at end of 24th second is to be determined. Choose Δt = 2 seconds.

t	T_{sk}	h	T_{BL}	G	f	f x Δt	Operation
20	519	0.0103	544	0.6	0.403	q_1=0.806	$\frac{1}{2}(q_1+q_4)$ = 1.618
21	519.4	.0106	568	.6	.834	q_2=1.668	q_2+q_3 = 3.324
21	519.8	.0106	568	.6	.828	q_3=1.656	Sum = 4.942
22	520.7	.0107	590	.6	1.215	q_4=2.430	q = 1.644
22	520.6	.0107	590	.6	1.217	q_1=2.434	$\frac{1}{2}(q_1+q_4)$ = 3.299
23	521.8	.0108	619	.6	1.728	q_2=3.456	q_2+q_3 = 6.886
23	522.3	.0108	619	.6	1.715	q_3=3.430	Sum = 10.185
24	524.0	.0109	640	.6	2.082	q_4=4.164	q = 3.395
24	524.0						

Therefore, the skin temperature at the end of 24th second is 524° F absolute. Notice that a constant value of G is used in the above computation. For a more accurate analysis the value of G should be based on T_{sk} given.

To improve the result, the following method, as given by Runge and Kutta, can be used for corrections. Instead of Δt = 2 seconds, table 3 is repeated with Δt = 4 seconds.

t	T_{sk}	h	T_{BL}	G	f	$f \times \Delta t$	Operation
20	519	0.0103	544	0.6	0.403	$q_1=1.612$	$\frac{1}{2}(q_1+q_4) = 4.976$
22	519.8	.0107	590	.6	1.230	$q_2=4.920$	$q_2+q_3 = \underline{9.704}$
24	521.6	.0107	590	.6	1.196	$q_3=4.784$	Sum $= 14.680$
24	523.8	.0109	640	.6	2.085	$q_4=8.340$	$q = 4.893$
24	523.9						

The correction is then given by

$$\xi = \frac{1}{15}(524 - 523.9) = 0.007$$

and the corrected skin temperature at the end of 24th second is

$$T_{sk} = 524 + \xi = 524.007$$

Runge and Kutta's method can also be applied to simultaneous equations of the type (B1) or to differential equations of higher order. For details, see reference 12.

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Table I

Variation of T_{BL} and h with time

①	②	③	④	⑤	⑥	⑦	⑧	⑨
t sec	H, ft Given	V_1 $\frac{ft}{sec}$ Given	T_1, °F Abs fig. 3	$T_{BL}-T_1$ fig. 5	T_{BL} ④+⑤	h' fig. 7	F_H fig. 10	$h = F_\beta F_\ell F_H h'$

Given β = total nose angle =

ℓ = nose surface length, ft. =

F_β (figure 8) =

F_ℓ (figure 9) =

Table 2

Numerical solution of more accurate transient skin temperature equation

Initial condition $t_o =$ sec $\qquad$ time interval $\Delta t =$

$T_o =$ °F Abs.

t	T_{sk}	h Table I	T_{BL} Table I	G	f	$f \times (\Delta t)$	Operation
t_o	T_o					q_1	$\frac{1}{2}(q_1 + q_4) =$
$t_o + \frac{\Delta t}{2}$	$T_o + \frac{q_1}{2}$					q_2	$q_2 + q_3 =$ ____
$t_o + \frac{\Delta t}{2}$	$T_o + \frac{q_2}{2}$					q_3	sum $=$
$t_o + \Delta t$	$T_o + q_3$					q_4	$q = \frac{1}{3}(\text{sum})$

$$t' = t_o + \Delta t, \qquad T' = T_o + q$$

If t', T' are used as initial conditions and the above table repeated, the skin temperature corresponding to $t = t_o + 2\Delta t$ can be obtained.

Table 3

Numerical solution of the simplified transient skin temperature equation

	n	1	2	3	4	etc.
1	t					
2	h (Table I)					
3	h/G					
4	$\frac{1}{2}[③_n + ③_{n-1}]$	—				
5	$\Delta t = [①_n - ①_{n-1}]$	—				
6	④ × ⑤	—				
7	$\sum_1^n ⑥$	0				
8	$e^{⑦}$					
9	T_{BL} (Table I)					
10	③ × ⑧ × ⑨					
11	$\frac{1}{2}[⑩_n + ⑩_{n-1}]$	—				
12	⑤ × ⑪	—				
13	$\sum_1^n ⑫ + T_i$	T_i				
14	$T_{sk} = ⑬/⑧$					

G = heat absorption capacity of skin, $\frac{\text{Btu}}{(\text{sq ft})(^\circ\text{F})}$

T_i = initial temperature of skin, °F abs

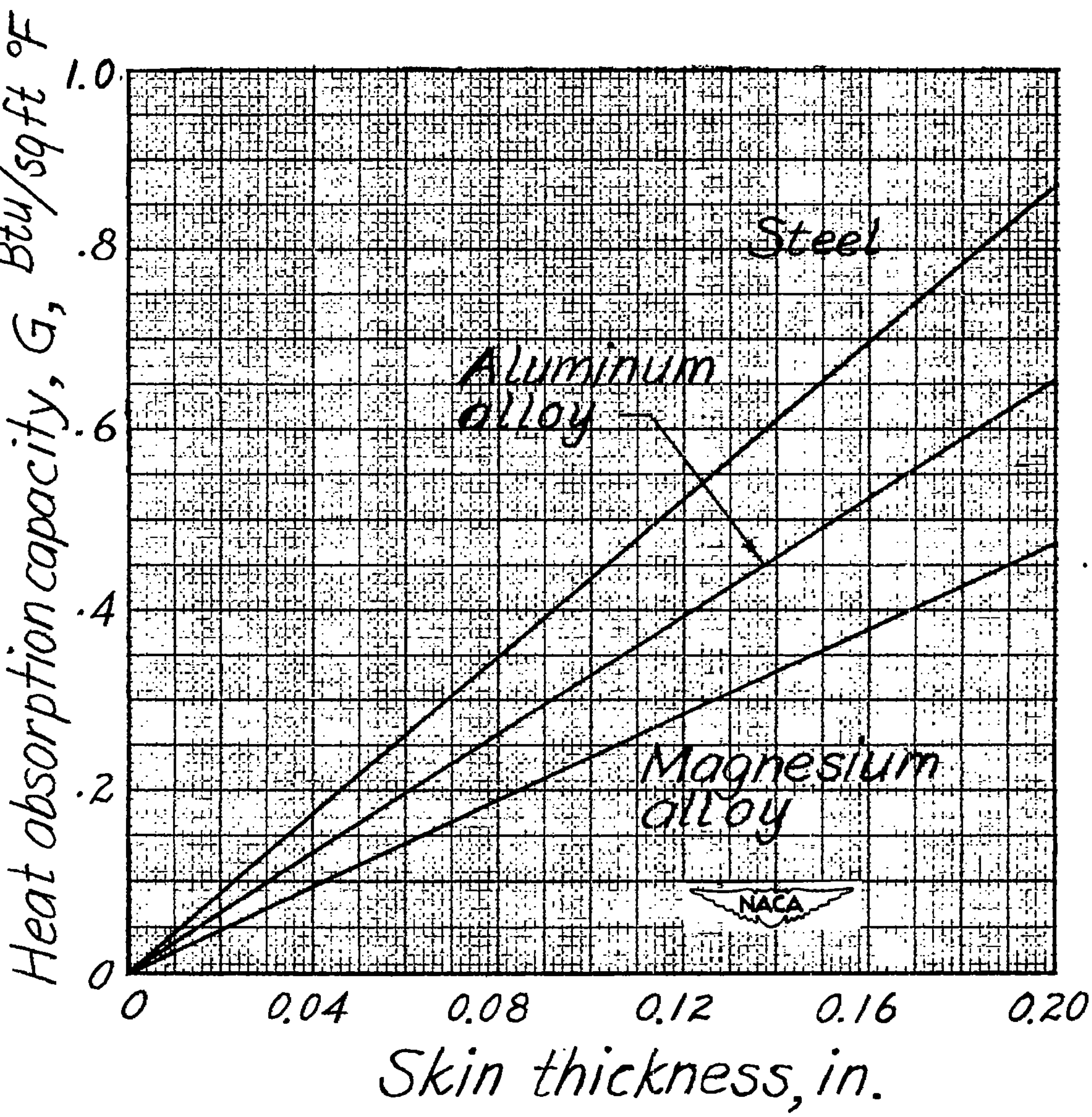


Figure 1.- Heat absorption capacity of various metal skins at room temperature (520 °F abs).

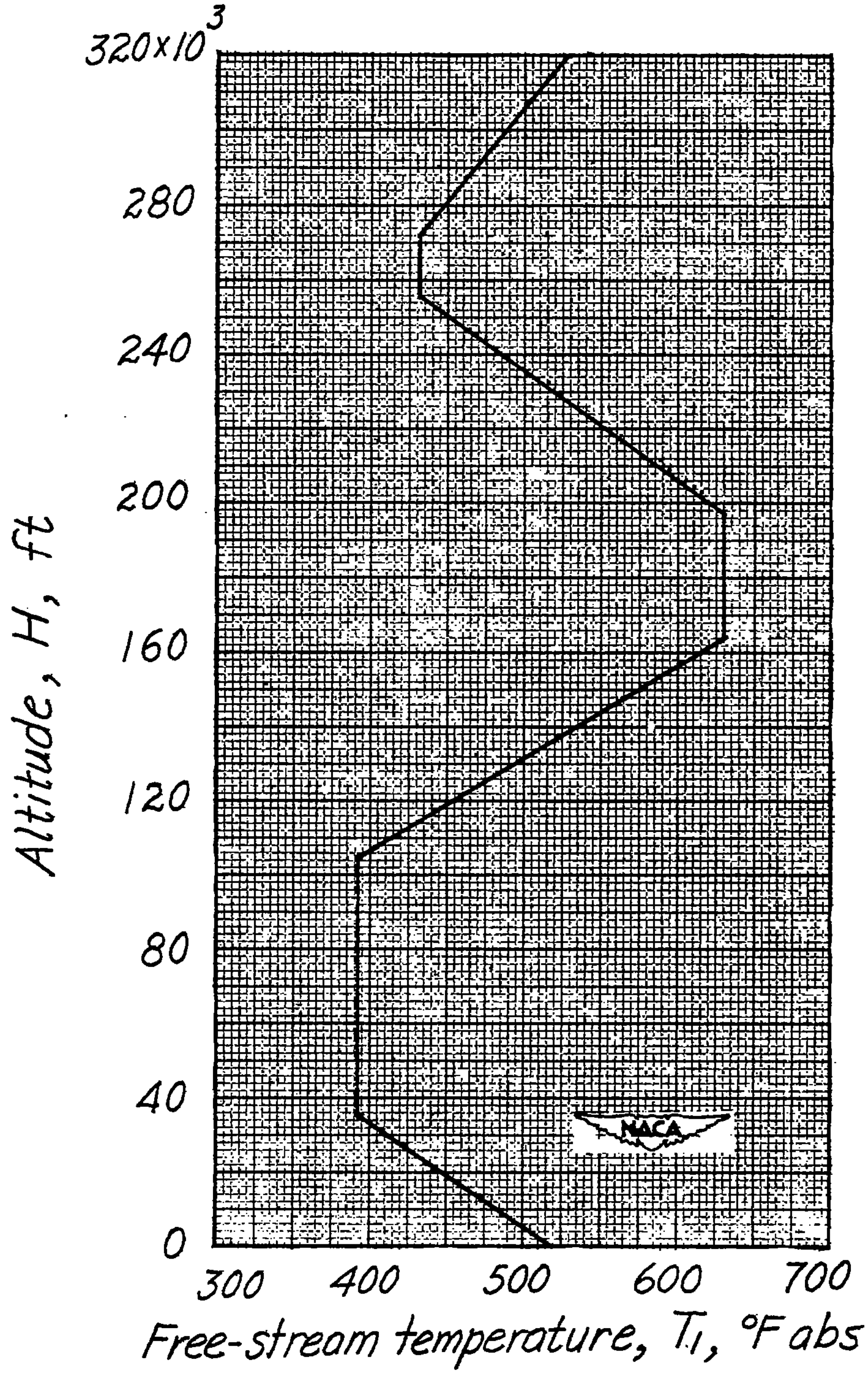


Figure 2.- Free-stream temperature at various altitudes (data obtained from reference 8).

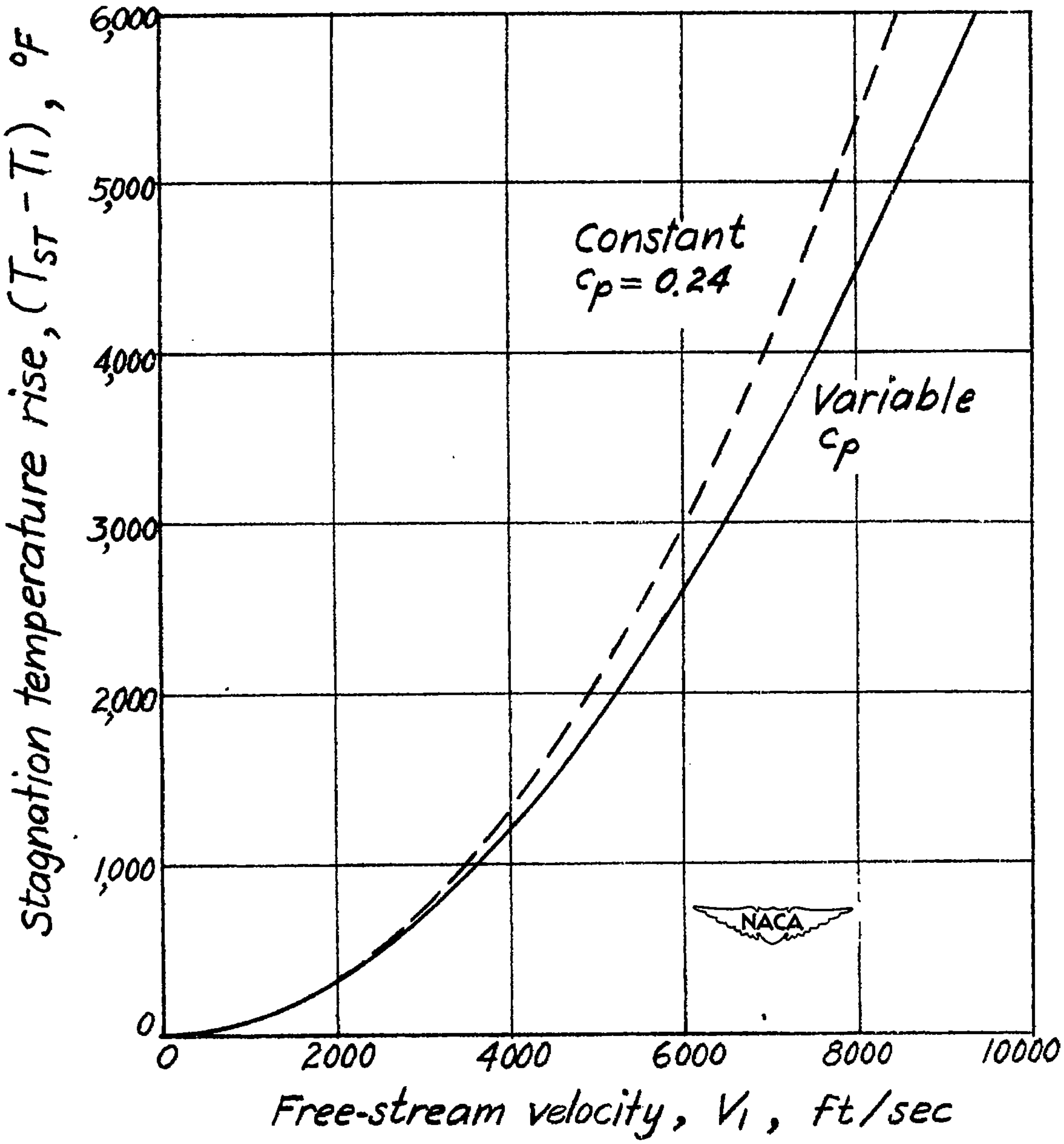


Figure 3.- Stagnation-temperature rise for both constant and variable c_p.

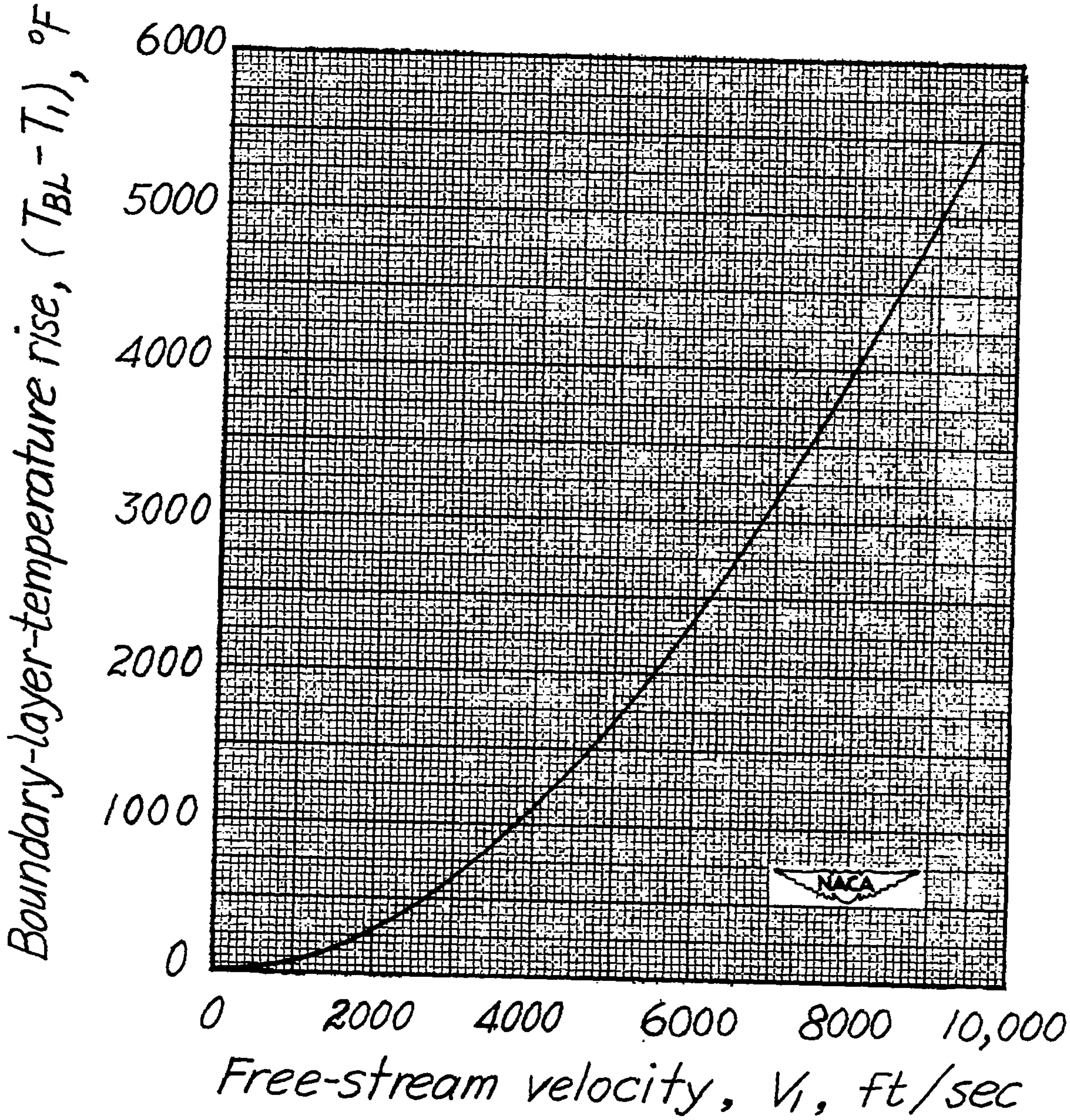


Figure 4.- The boundary-layer-temperature rise at various velocities (variable c_p).

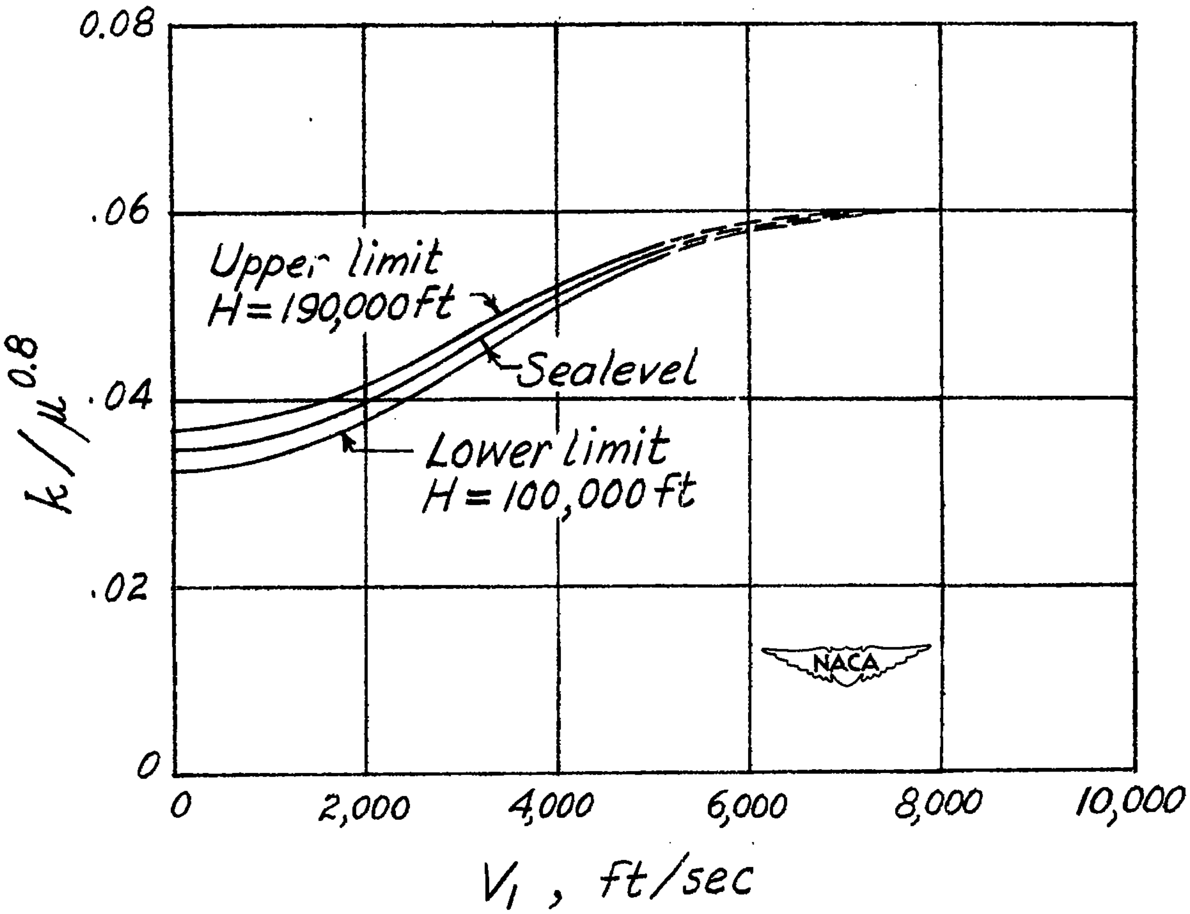


Figure 5.- Variation of $(k/\mu^{0.8})$ with velocity.

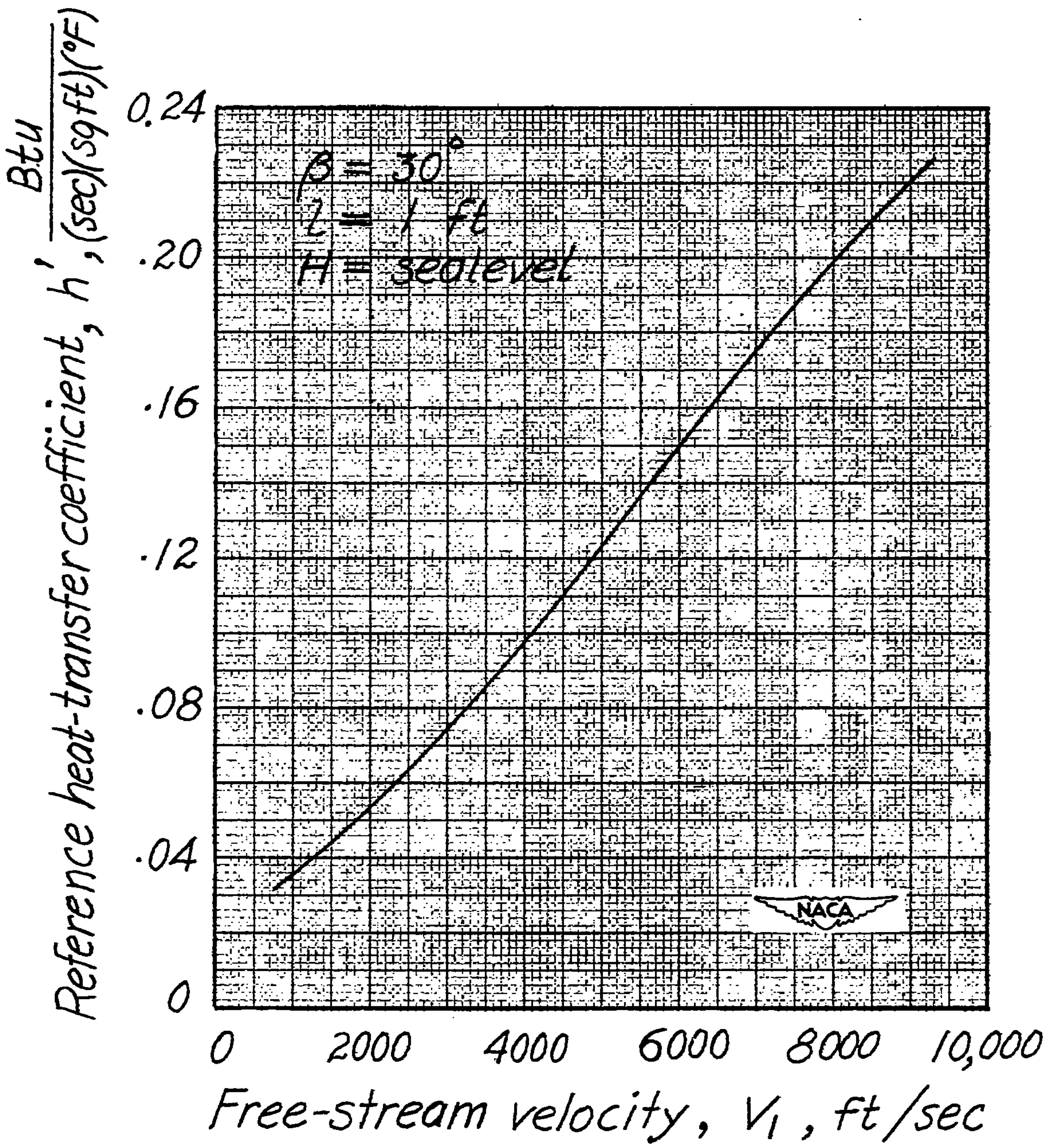


Figure 6.- Variation of reference heat-transfer coefficient with velocity.

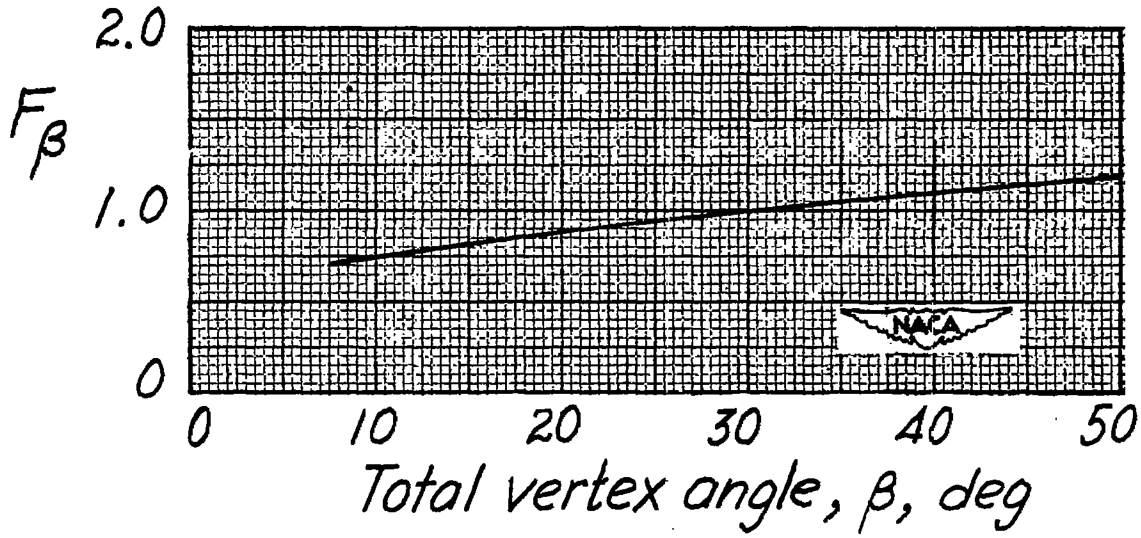


Figure 7.- Correction factor for β.

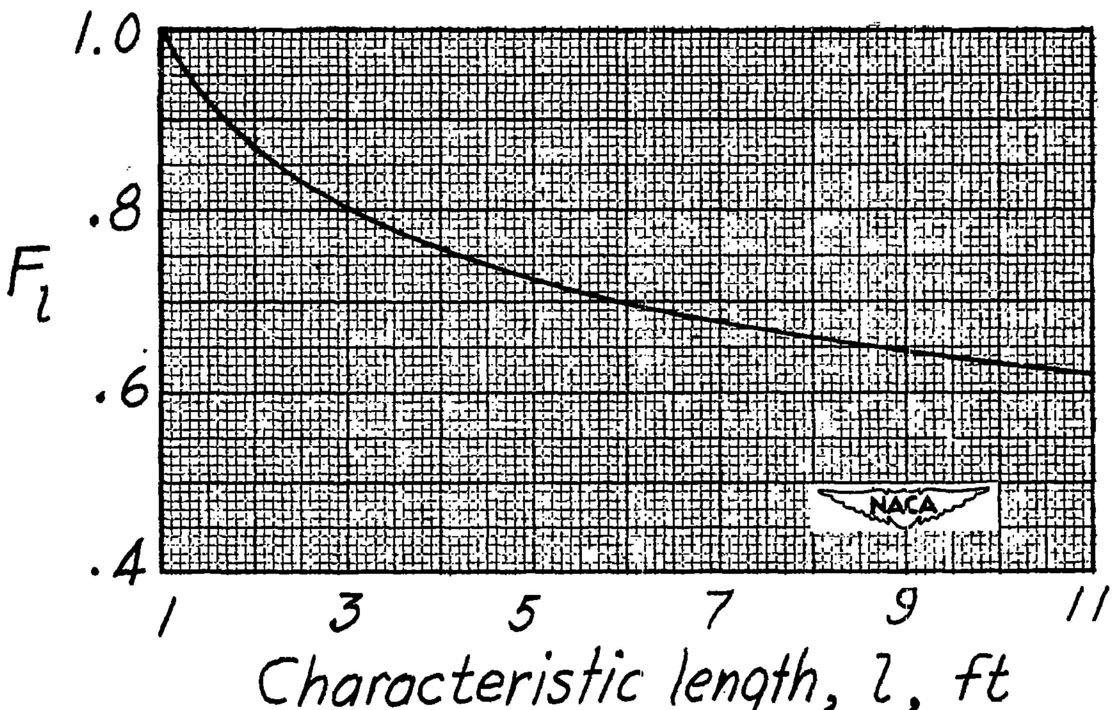
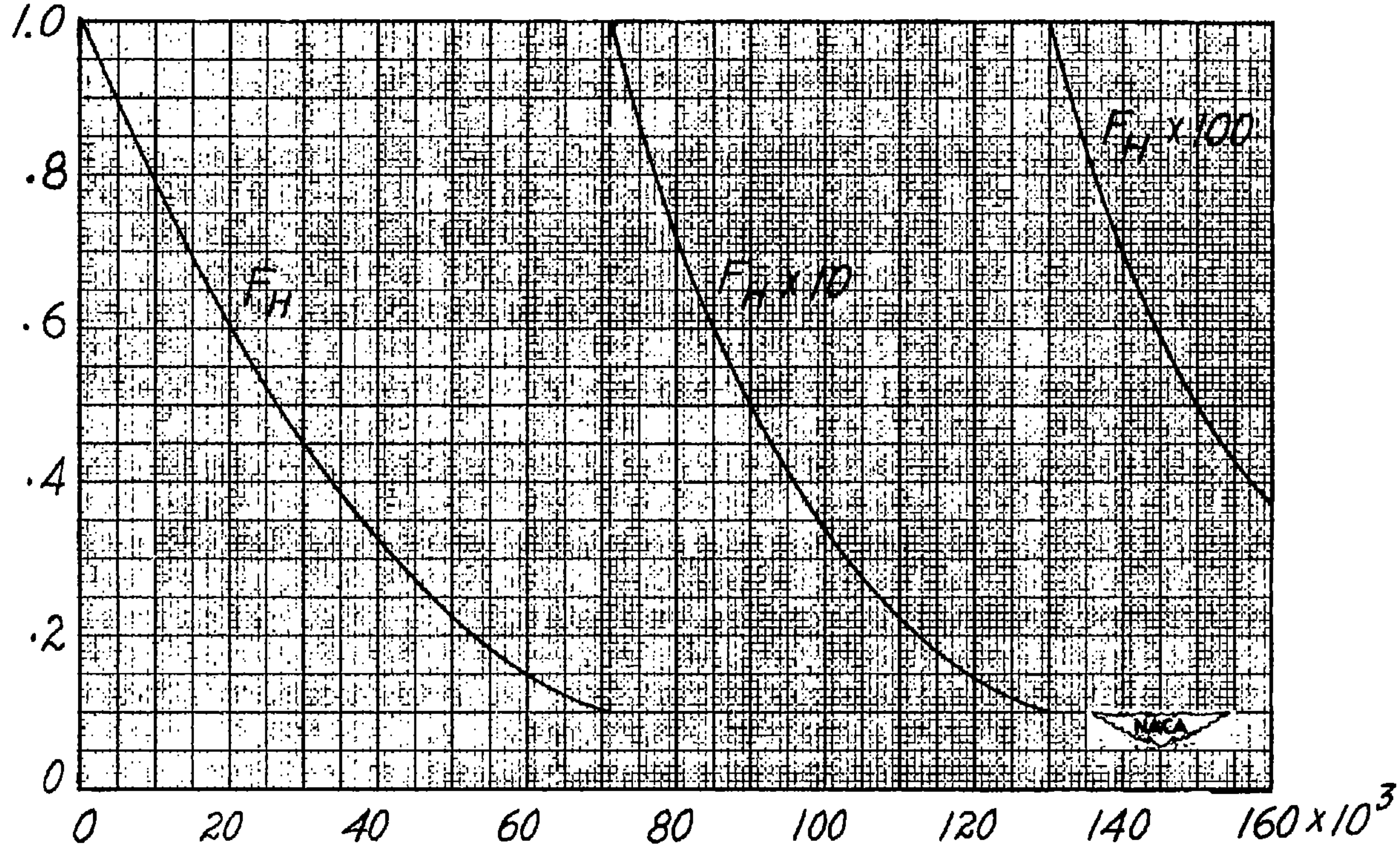
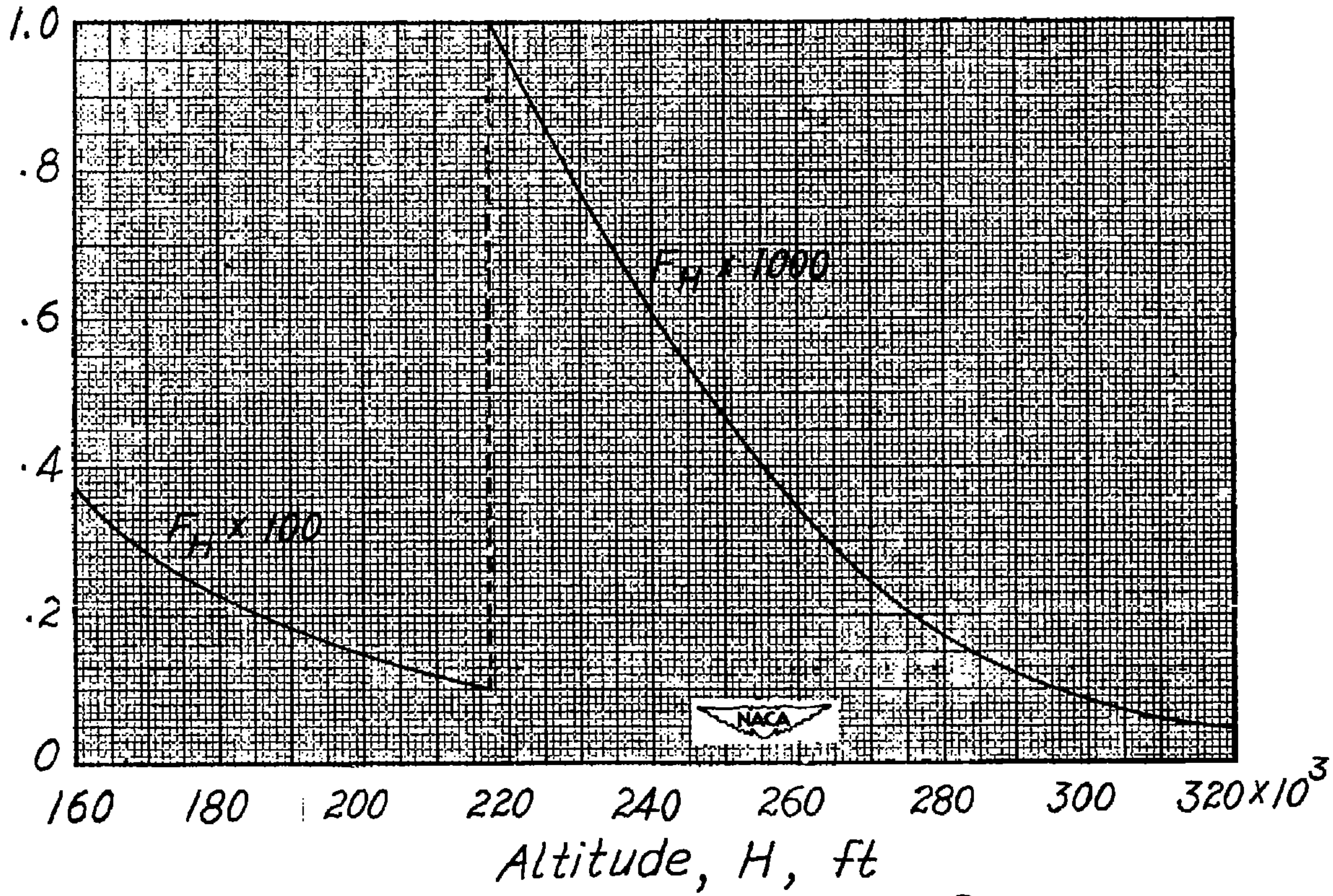


Figure 8.- Correction factor for l.



(a) From sealevel to 160,000 ft.

Figure 9. - Correction factor for altitude.



(b) From 160,000 ft to 320,000 ft.

Figure 9.- Concluded.

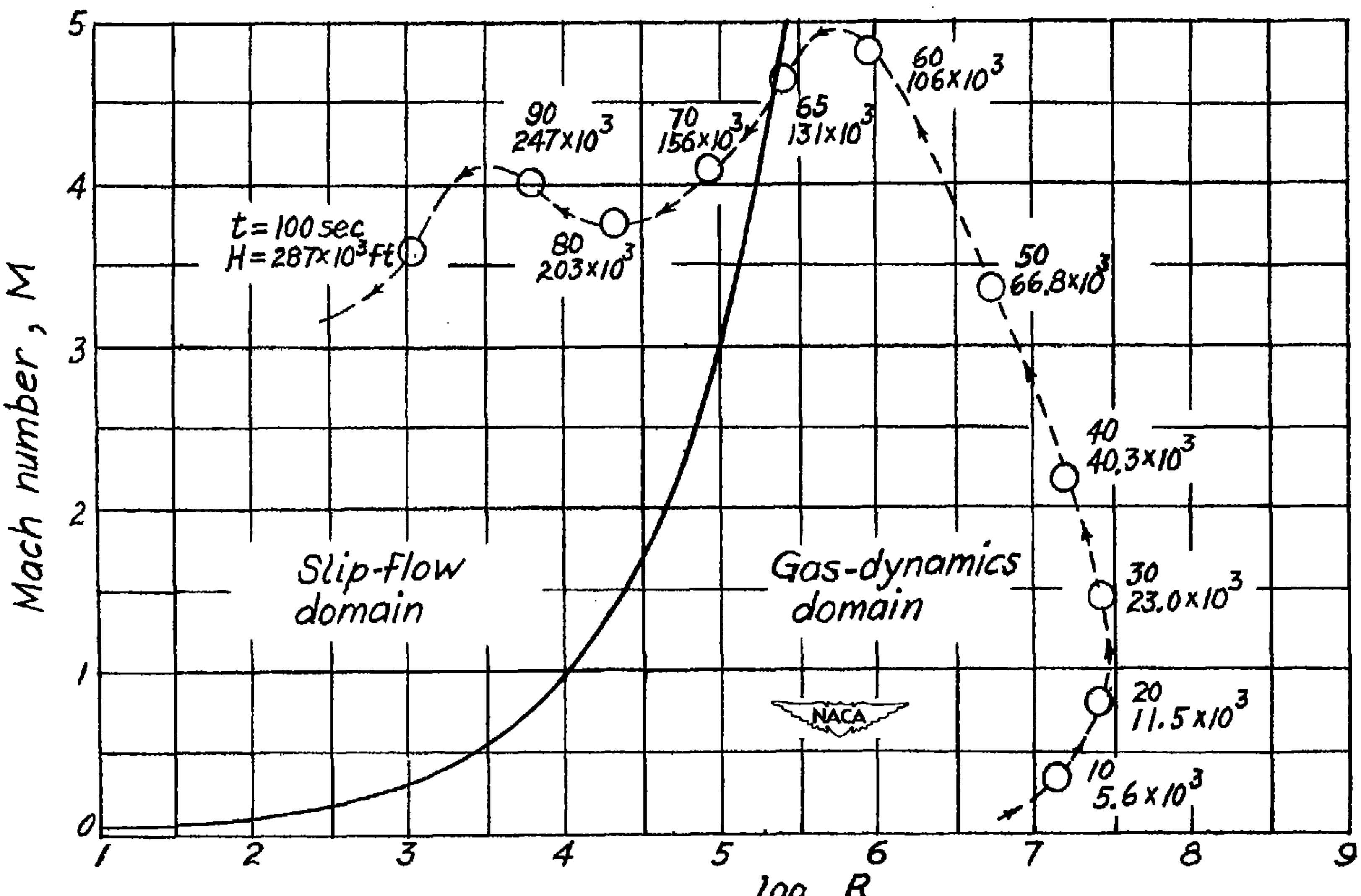


Figure 10.- Different domains of flow conditions entered by V2 missile 21, fired on March 7, 1947.

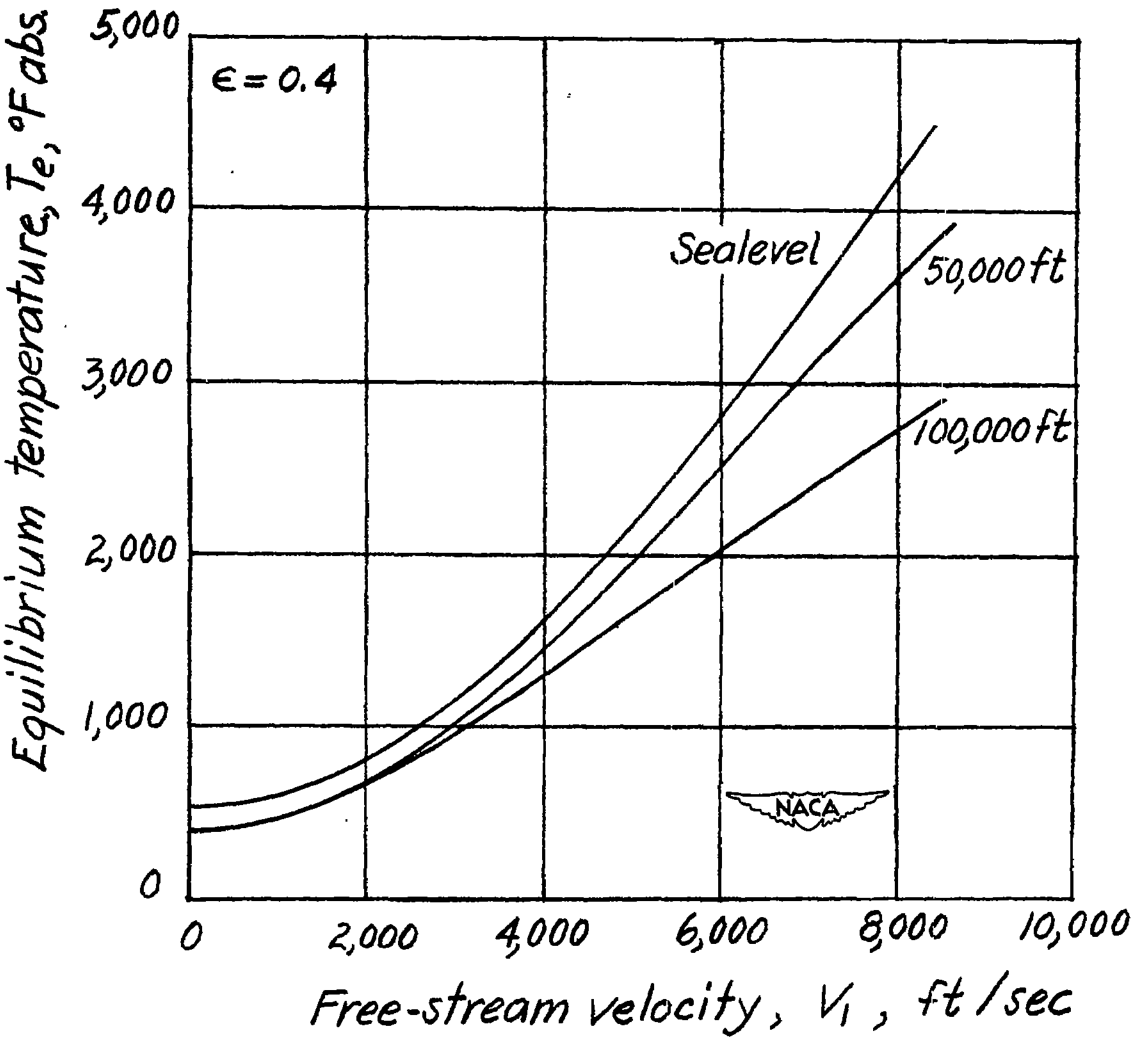


Figure 11.- Equilibrium temperature at various velocities and altitudes.

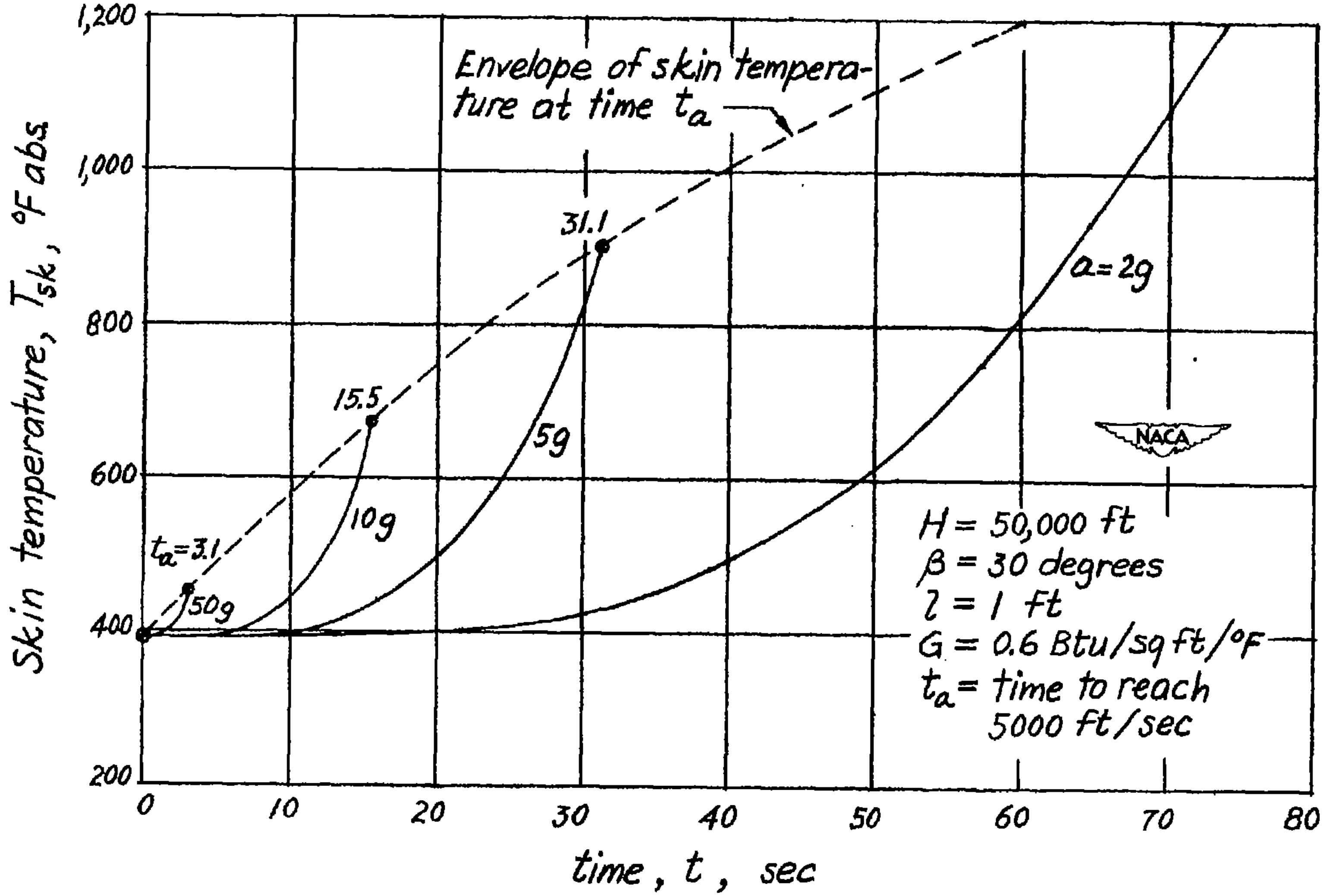


Figure 12.- Time history of skin temperature for uniformly accelerated body at constant altitude.

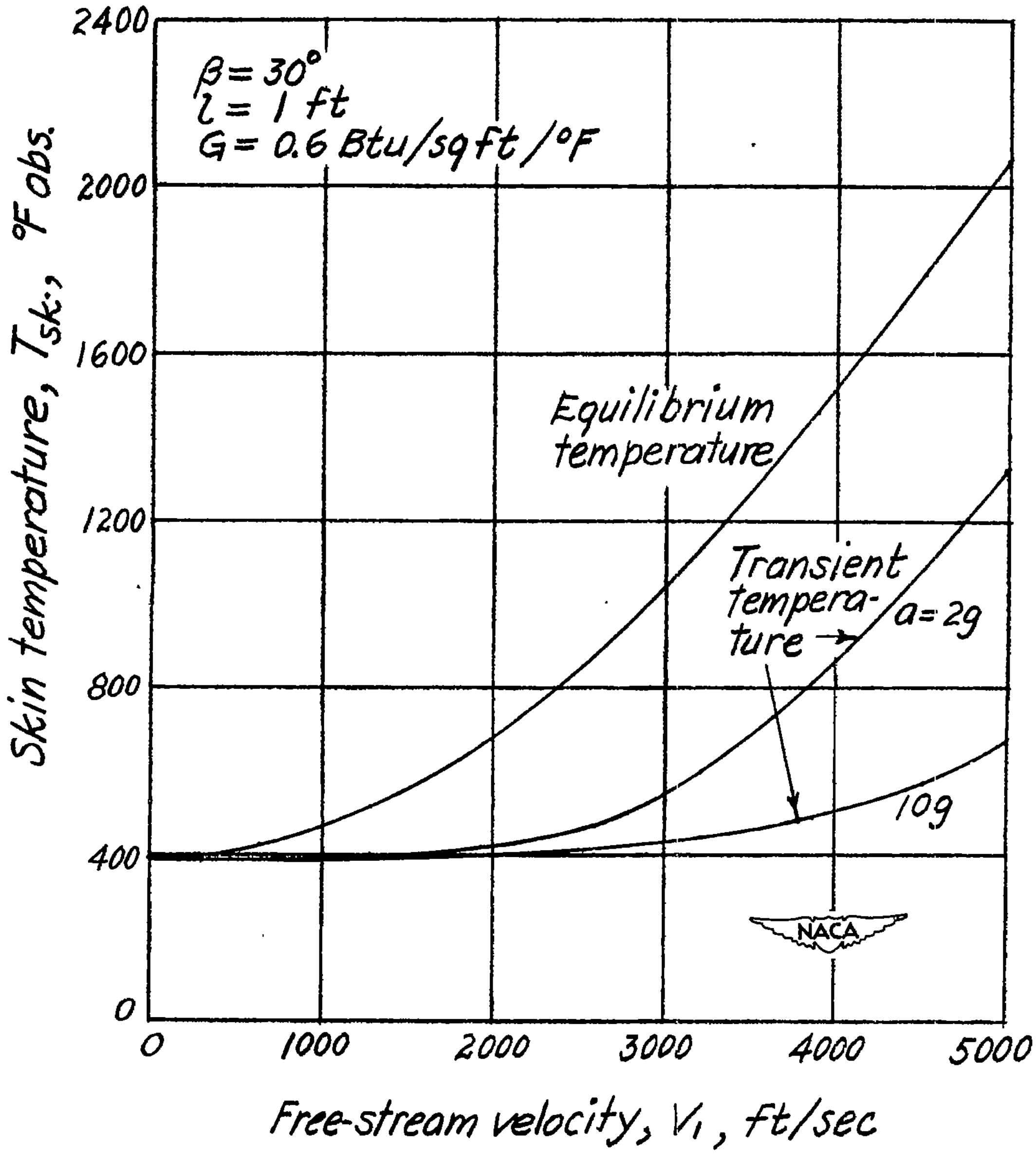


Figure 13.- Comparison of transient and equilibrium temperature for uniformly accelerated body at altitude 50,000 ft.

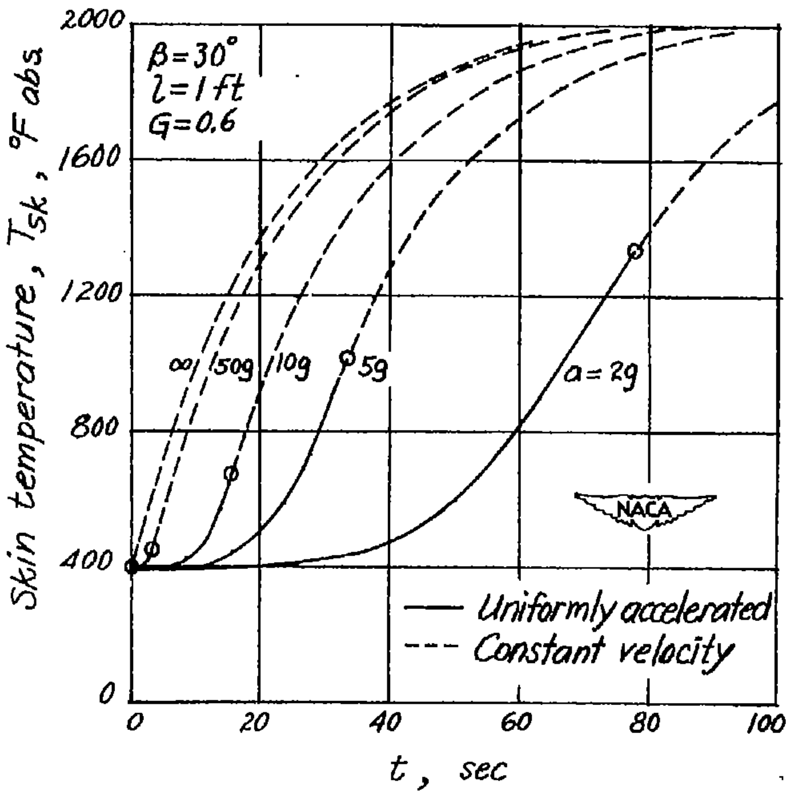


Figure 14.- Time history of uniformly accelerated body at altitude 50,000 ft. Constant velocity after the body reaches 5000 ft/sec.

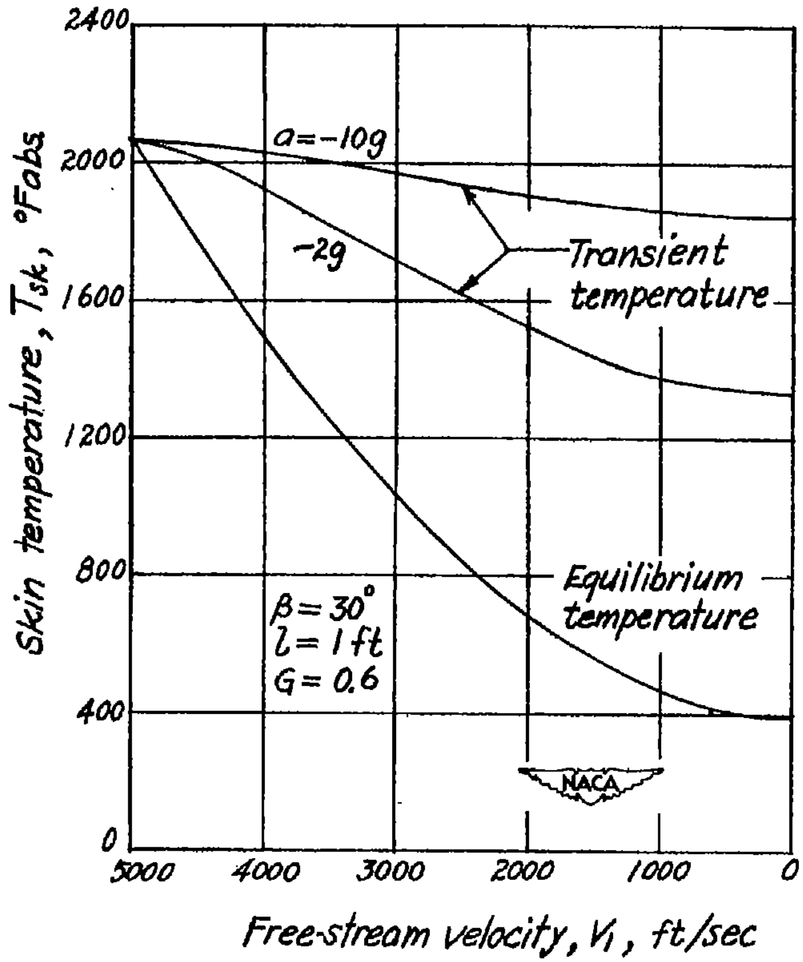


Figure 15.- Skin temperature of uniformly decelerated body at constant altitude 50,000 ft.

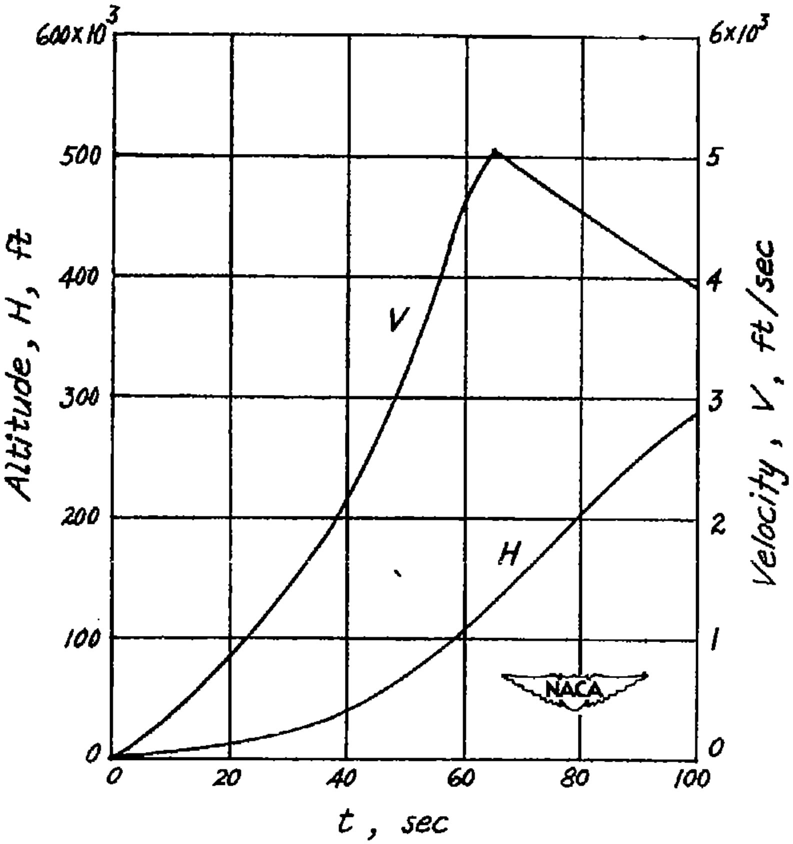


Figure 16.- Velocity and altitude diagram for V2 #21, fired on March 7, 1947 (data obtained from Naval Research Laboratory).

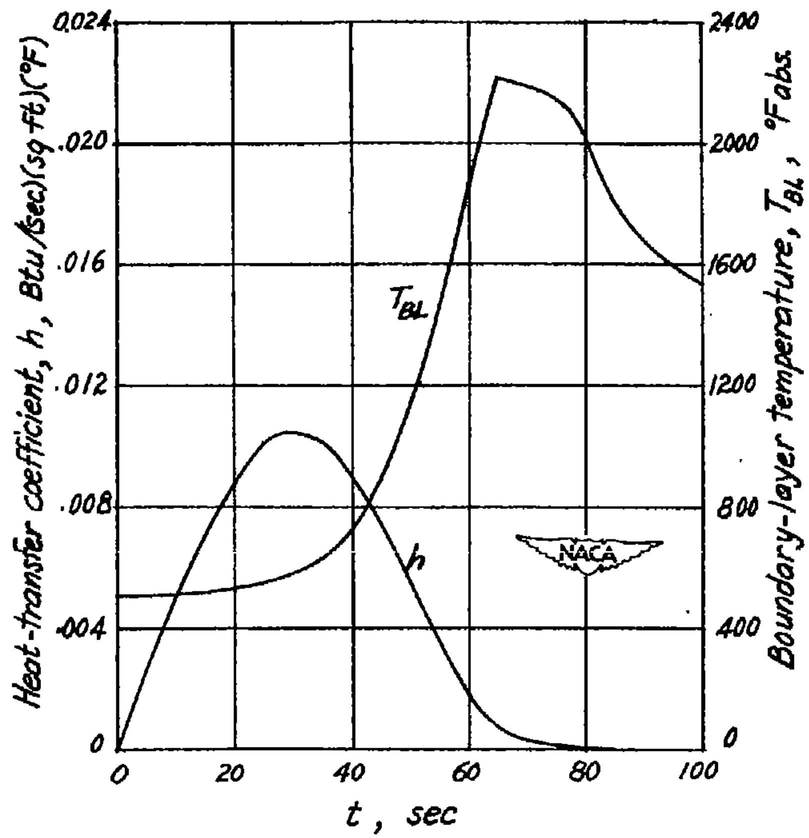


Figure 17.- Variation of T_{BL} and h with time, example 1.

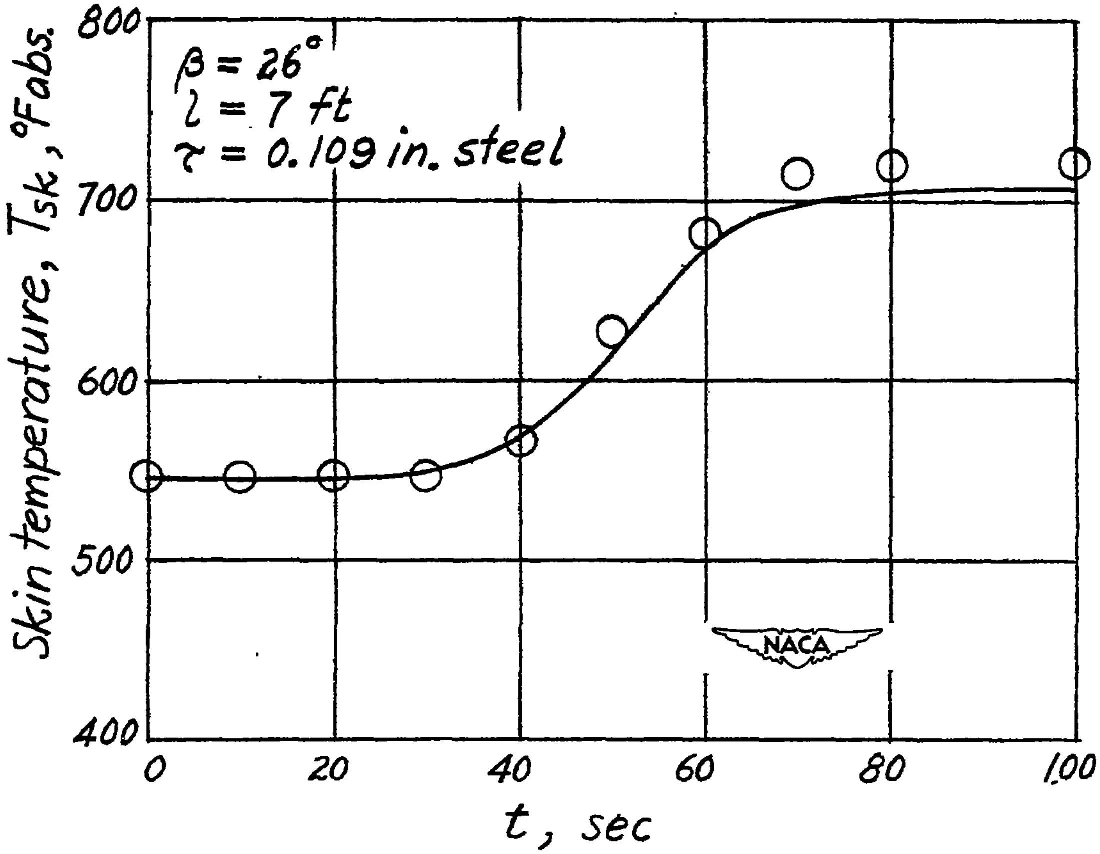


Figure 18. - Comparison of calculated and measured skin temperatures for V2 #21, fired on March 7, 1947 (measured data obtained from Naval Research Laboratory).

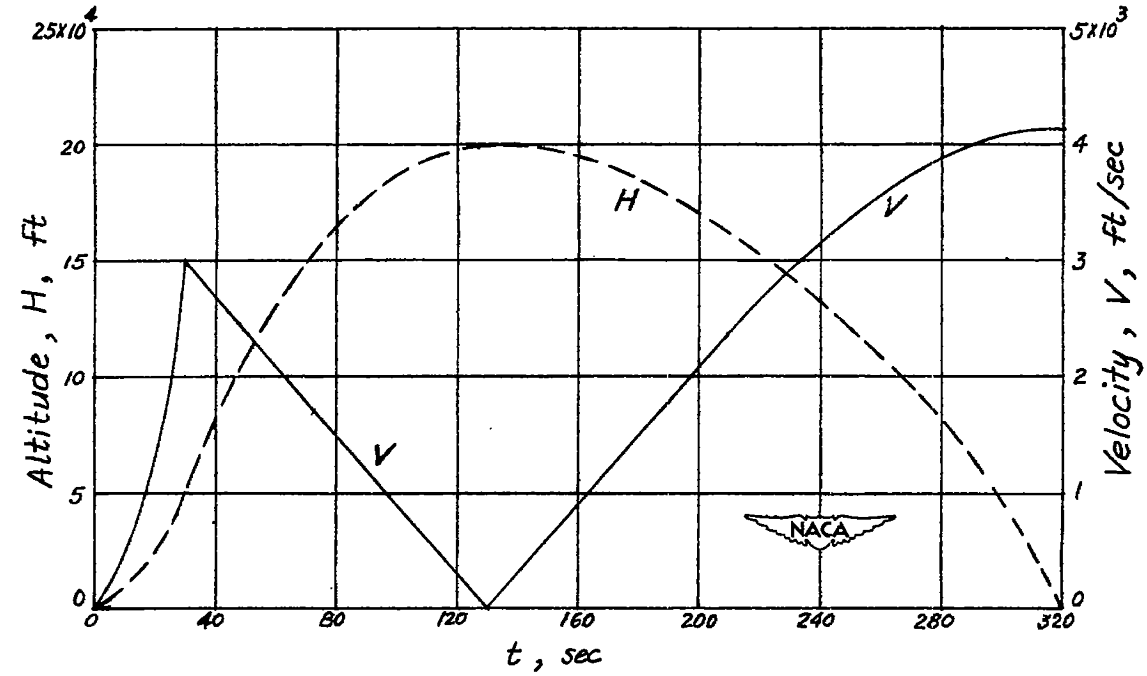


Figure 19.- Assumed velocity and altitude diagram for example 2.

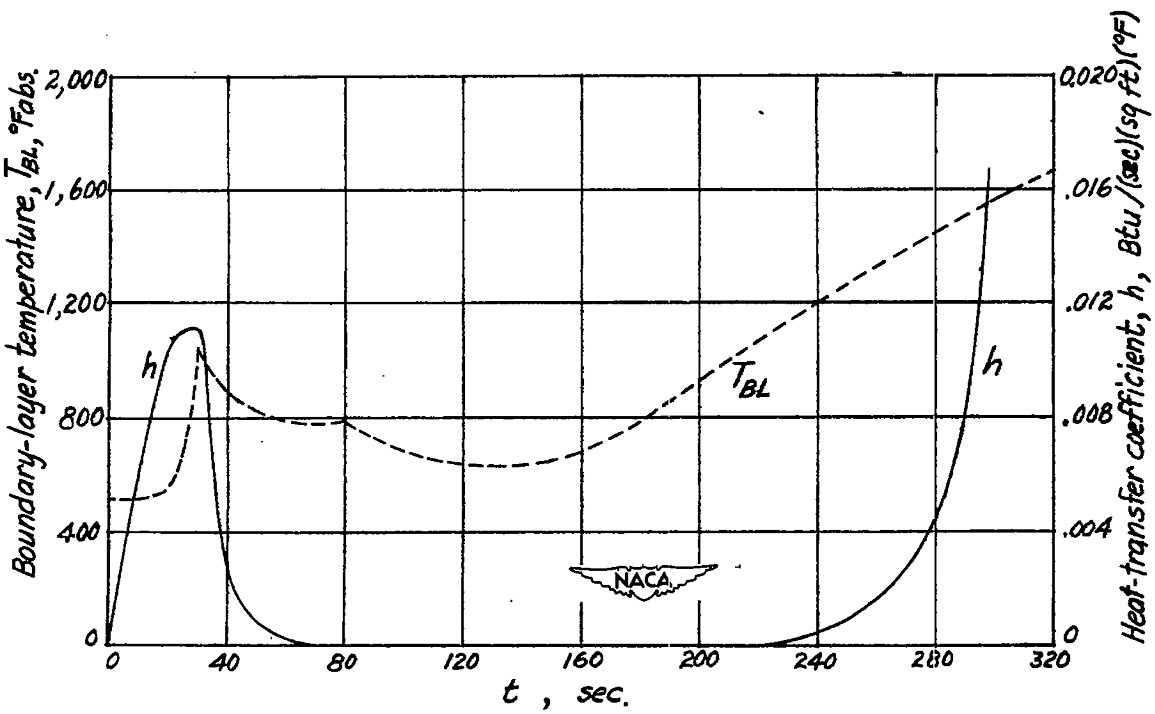


Figure 20.- Variation of T_{BL} and h with time, example 2.

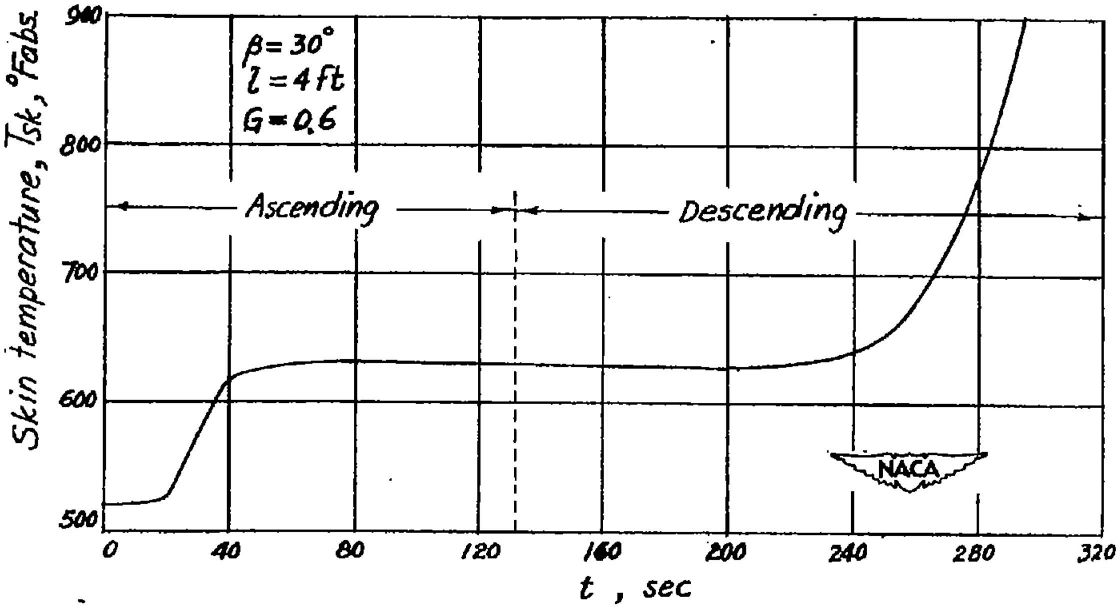


Figure 21. - Variation of skin temperature with time, example 2.

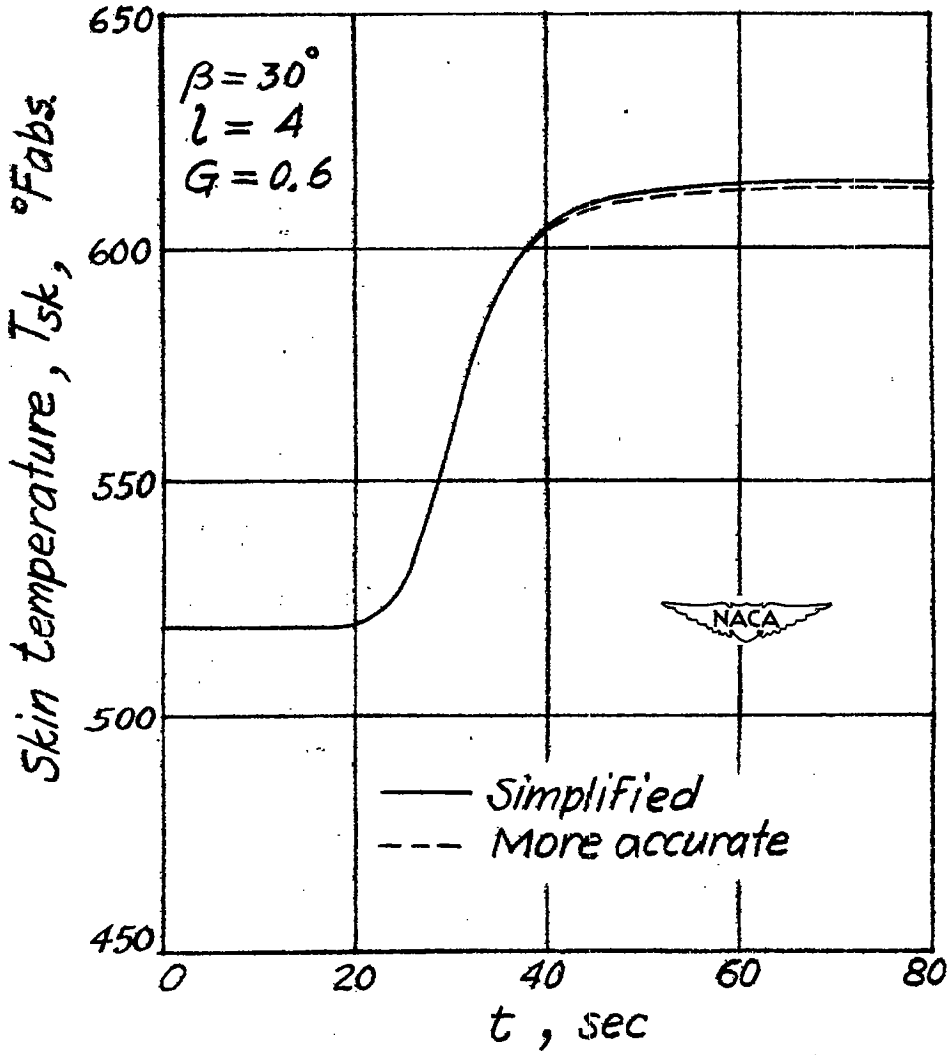


Figure 22.- Comparison of simplified method and the more accurate method in example 3.

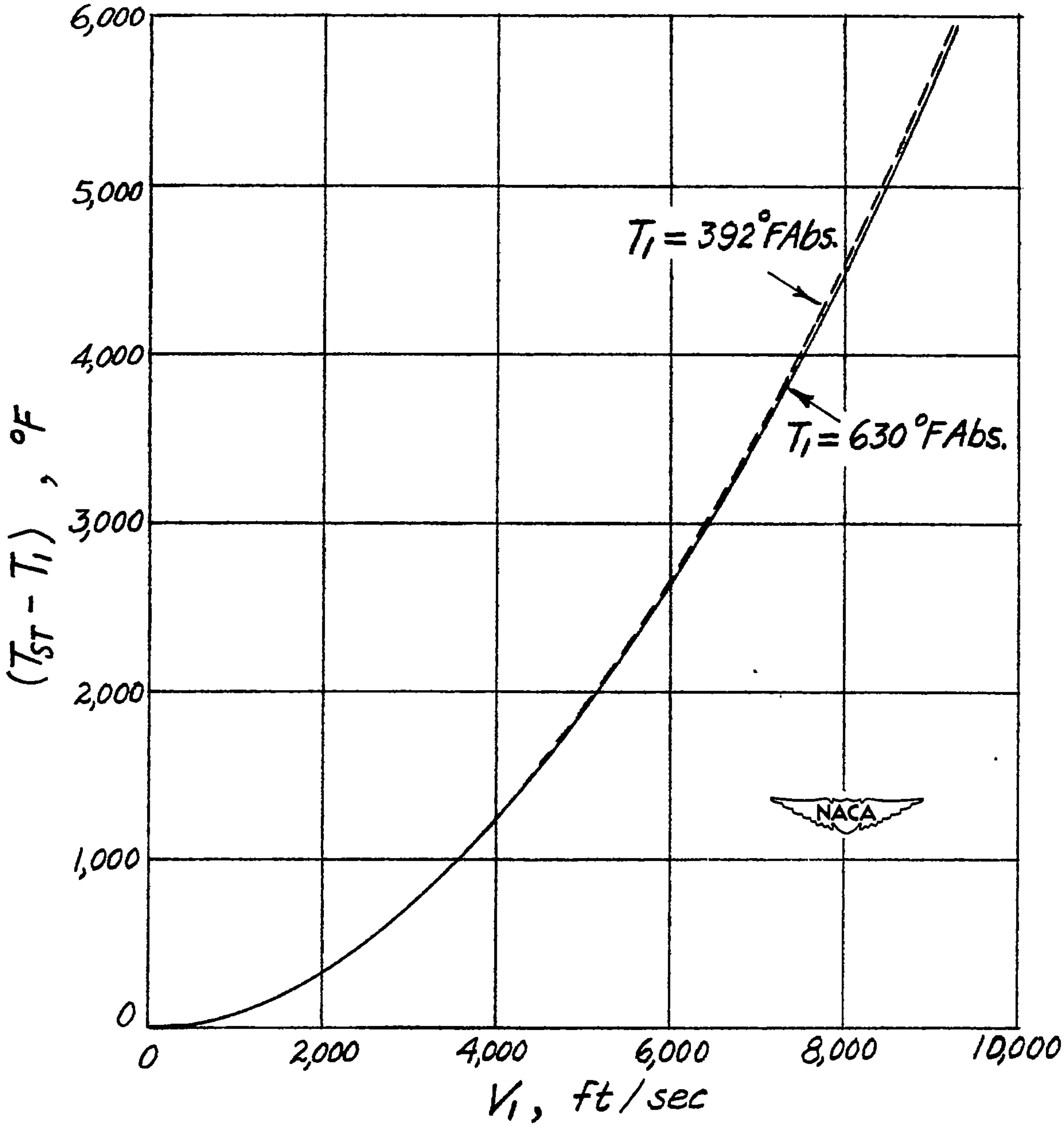


Figure 23.- Stagnation-temperature rise; variable c_p.